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This report summarizes the feasibility of, and scaling relationships for, centrifugal modelling of the GLCM shelter. Similitude between model and prototype is deemed possible if constitutively similar reinforced concrete and soil are used in the model. However, currently available US centrifuges limit GLCM shelter model scales to 1/150 to 1/300, which causes difficulties in model construction with microconcrete and miniaturized reinforcement. Therefore, centrifugal modelling of GLCM components, coupled with computer analyses, is recommended.

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PREFACE

This report was prepared by the Department of Civil Engineering, University of Florida, Gainesville FL 32511, under Contract Number F08635-83-C-0136, Task No. 83-1, for the Air Force Engineering and Services Center, Engineering and Services Laboratory (HQ AFESC/RD), Tyndall Air Force Base, Florida. This report is published as submitted to the University of Florida by Captain Dale M. Bradley, as his Master of Engineering Thesis, under the direct supervision of Professors F. C. Townsend, F. E. Fagundo, and J. L. Davidson. Captain Paul L. Rosengren, Jr., was the HQ AFESC/RDCS Project Officer.

This report summarizes work performed between July and December 1983 and discusses the feasibility of, and scaling relationships for, centrifugal modelling of the GLCM shelter. Similitude between model and prototype is deemed possible if constitutively similar reinforced concrete and soil are used in the model. However, currently available US centrifuges limit GLCM shelter model scales to 1/150 to 1/300, which caused difficulties in model construction with microconcrete and miniaturized reinforcement. Therefore, centrifugal modeling of GLCM components, coupled with computer analyses, is recommended.

This report has been reviewed by the Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication.

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SECTION I

INTRODUCTION

A. GENERAL

Appropriate load and response testing is fundamental to design of a new structure. This type of evaluation is done commercially with component testing and analytical computer simulation. A much more intensive and comprehensive testing program is needed for protective structures developed for the Department of Defense. The degree of uncertainty must be quite narrow to satisfy defense design engineers. Simple application of safety factors is not sufficient and the entire structure should be tested under explosive loading. This often requires full-scale testing of structures over several geological sites to accurately evaluate the design assumptions and computer models. This has been done in the past, but at monumental cost and substantial safety risk.

To reduce testing costs and safety risks, the Department of Defense decided to test scaled models on the order of one-half to one-twentieth the size of the prototypes. While this order of scaled testing reduced costs, the reduction has not been enough to meet budget criteria. Additionally, there are some technical concerns regarding testing smaller than one-tenth the prototype size. Murphy (Reference 1) suggests that the dead load effects of the structure itself are not appropriately handled in one gravity testing. Since dead loads play a substantial role in dynamic response, this type of scaled testing is limited.

Faced with these concerns, the Air Force has started to investigate the possibility of using a centrifuge to test its new Ground Launch Cruise Missile (GLCM) shelter, (Figure 1). This structure is 278 feet long, 164 feet wide and 38 feet tall. With present centrifuges the GLCM could be tested at 1/150 to 1/300 the size of the prototype with explosives reduced by $1/n^3$, where n is the number of gravities (Reference 2). Coupled with data provided by Mikasa and Takada (Reference 3) which indicate that the centrifuge appropriately models dead load effects, the centrifuge shows substantial potential for cost reduction. However, major concerns prevail in the community regarding the use of the centrifuge.

These major concerns are constructability of such a small model and applicability of model data to prototype response. This report will evaluate the scaling laws that will ensure the similitude between model and prototype. To the extent possible, the formulated scaling laws will indicate the feasibility of the centrifuge for GLCM shelter testing.

B. OBJECTIVE

The primary objective of this effort is to investigate the feasibility of using a centrifuge to test scaled models of the GLCM shelter. The basic premise of the study is to see if a scaled GLCM shelter can be tested in a centrifuge in such a manner that ensures similitude between model and prototype. In other words, can the generated data of a centrifuge model testing program be used to predict the GLCM shelter response. To meet this objective, the following must be completed:

1. All relevant parameters of the structure and surrounding material that affect structure response due to explosive loading must be identified.

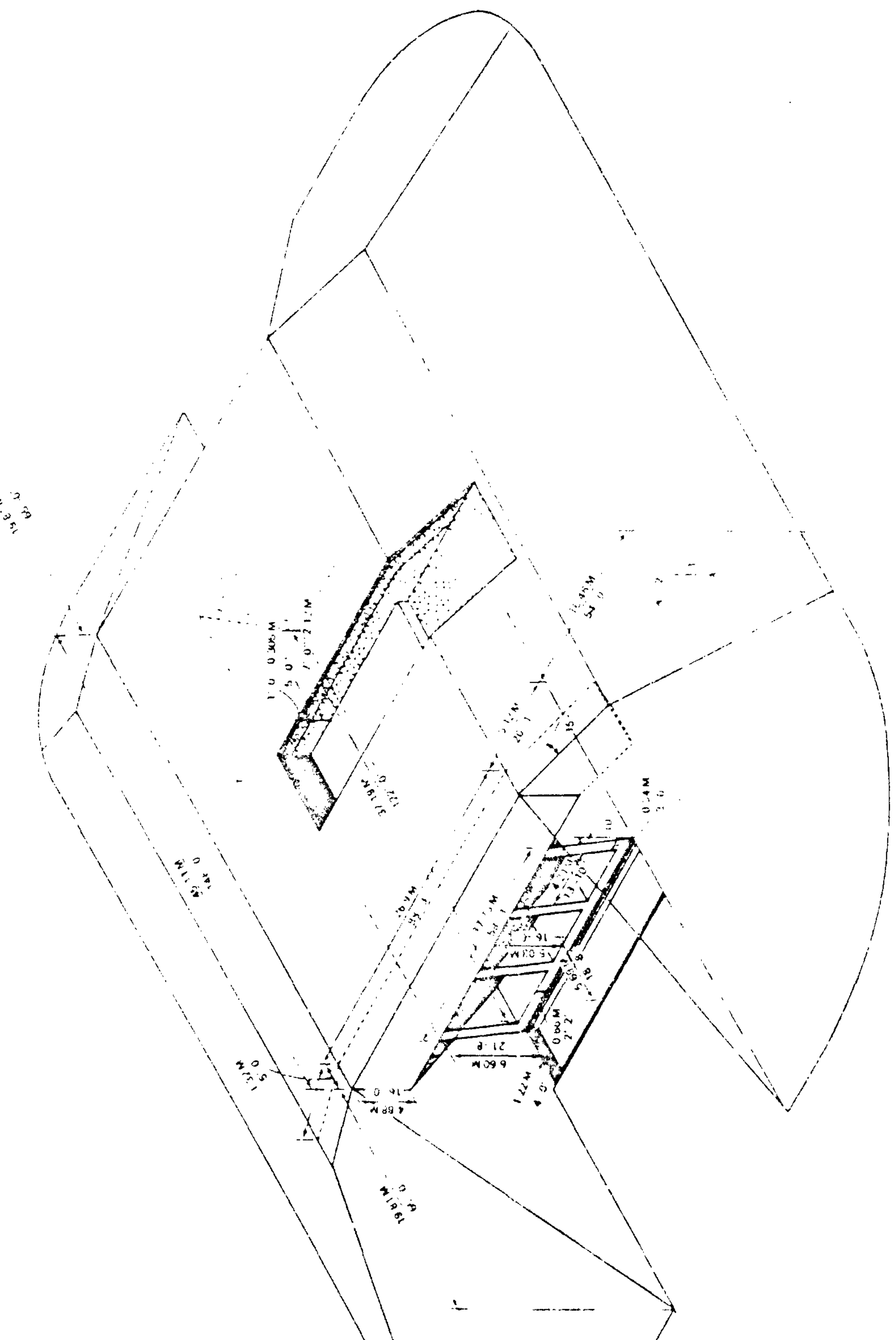

FIGURE 1. GROUND LAUNCH CRUISE MISSILE SHELTER

2. Using the response parameters, formulate scaling laws that will relate the model to the prototype.
3. Based on the scaling laws, determine the physical limitations of the model design and testing procedure.
4. Based on physical limitations, recommend the model scale and centrifuge to be used in a testing program.

C. SCOPE

The effort began with a review and study of theory of modelling techniques and procedures. The review included simple to complex problems resulting in the identification and use of the Buckingham Pi Theory. The theory's application is discussed and demonstrated in a generic problem and used to prove a common table of scaling laws. This established the methodology to develop the GLCM centrifugal scaling laws. The response parameters were then identified and used to establish the system (independent) parameters which would affect them due to the loading type and materials involved. The research progressed into a trial and error procedure, using the Buckingham Pi Theory to identify a complete set of scaling laws to govern the model testings. Based on the literature, review qualifications and limitations to these laws are addressed and clarified based on the extent that they may cause perturbations to similitude. A comparison of the available centrifuges in the United States to the physical size of the GLCM shelter was then conducted to evaluate possible scales to be used. This resulted in a list of potential scales and centrifuges to be matched together in a model testing program. The effort concludes with suggestions for explosives, instrumentation, and materials to be used in the model.

D. BACKGROUND

Practically all the literature dealing with scaling uses the Buckingham Pi Theory for the basis of their study, regardless if it is in a one- or multiple-gravity environment. A substantial list of references deals with scaling laws associated with one-gravity tests (see Reference List); however, dynamics centrifuge research is practically nonexistent. Several researchers (References 3,4,5) have studied structures over soil in multiple-gravity environments but only under static loads. Many others have studied dynamic loading on structures over soil but in one gravity (References 6,7). Apparently only a very few have studied the scaling laws in the condition of dynamic structure/soil interaction within a centrifuge (References 8,9).

Regardless of the type of experiments researched or conducted, the primary axiom of scaled testing is to use, when possible, the same material properties in the model as in the prototype. This drastically simplifies the development of the scaling laws. The axiom has resulted in the development of both microconcrete and gypsum mortar to satisfy scaled-down concrete mixes. For soils, simple reduction in particle size has proven unsatisfactory, due to the complex interrelationship of soil strength and response to particle size. Subsequently, researchers have used the exact same soil in model as in prototype to ensure consistency in material properties. Perturbations from this action have proven insignificant if the grain size is maintained much smaller than the scaled structure element (Reference 8). Previous researchers (Reference 9) feel that creditable scaling laws can be developed. This effort will concentrate on that endeavor, but the author premises such research with the questionable constructability of the complex GLCM shelter at any scale smaller than 1/100.

SECTION II

MODELLING THEORY

A. GENERAL

In the initial evaluation of models, it was necessary to define the function that relates all the parameters affecting the predicted phenomena (dependent parameter). If stress (q_1) is dependent on q_2, q_3, q_4 (system parameters) then there exists a function such that:

$$q_1 = f(q_2, q_3, q_4) \tag{1}$$

In simple systems in which the dependent parameter (q_1) is only dependent on a small group of parameters (q_2, q_3, q_4), the function is readily evaluated; however, in complex systems, such as the GLCM, the list of parameters is quite numerous, making the determination of the function practically impossible.

In 1914, E. Buckingham proposed a theory (known today as Buckingham Pi Theory) which eliminates the need to know the function and allows direct similitude through the use of π terms. Buckingham states that if the equation relating the dependent parameter to system parameters is a dimensionally homogeneous* it can be reduced to a complete set of dimensionless products (Reference 10). Therefore, if

$$q_1 = f(q_2, q_3, q_4, \ldots q_n) \tag{2}$$

---

* To be dimensionally homogeneous is to be completely defined by one set of relevant dimensions such as mass, length, and time.

it can be defined as

$$f(q_1, q_2, q_3, q_4, \ldots q_n) = 0 \tag{3}$$

For which the Buckingham Pi Theory states,

"If the equation $f(q_1, q_2, q_3, q_4 \ldots q_n) = 0$ is complete, the solution has a form $f(\pi_1, \pi_2, \pi_3, \pi_4, \ldots \pi_{n-k}) = 0$ where the π terms are independent products of the parameters q_1, q_2, etc., and are dimensionless in the fundamental dimensions" (Reference 11). In this case, k is the rank of the dimensional matrix.

Essentially, Buckingham is saying that if all the relevant parameters are included in the initial equation, then similitude between model and prototype can be achieved if

$$\pi_{n_{model}} = \pi_{n_{prototype}} \tag{4}$$

B. EXAMPLE

Langhaar (Reference 10) provides an illustrative example of the use of the Buckingham Pi Theorem on a hypothetical problem in which the dependent parameter to be evaluated is P. P is dependent on Q, R, S, T, U, & V (System Parameters). Where P, Q, R, S, T, U, & V are defined in terms of mass (M), length (L), and time (t) as

$$P = (M^2 L)^{K_1},\ Q = \left(\frac{t}{M}\right)^{K_2},\ R = \left(\frac{M^3}{L}\right)^{K_3},\ S = (t^3)^{K_4},$$

$$T = (L^2 t)^{K_5},\ U = \left(\frac{L}{M^2 t}\right)^{K_6},\ \text{and}\ V = (ML^2 t^2)^{K_7}$$

With all parameters defined in fundamental units M, L, t, a dimensional matrix is developed.

	P	Q	R	S	T	U	V
M	2	-1	3	0	0	-2	1
L	1	0	-1	0	2	1	2
t	0	1	0	3	1	-1	2

To ensure that the π terms are independent, the rows of the dimensional matrix must be linearly independent. If the rank (r) of the matrix is equal to the number of rows, independence is guaranteed. The size of any internal square matrix, whose determinant is nonzero, indicates that rank. This value determines the number of π terms (s) to be generated where

$$s = a - r \tag{5}$$

and a equals the number of parameters in the problem. The determinant of the last three columns (T, U, V) is

$$\begin{vmatrix} 0 & -2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 1 \tag{6}$$

Therefore,

1) The rank equals 3.

2) The rows are linearly independent.

3) The number of expected π terms is 4.

Since the π terms are products of the parameters they have the form of

$$\pi = P^{K_1} Q^{K_2} R^{K_3} S^{K_4} T^{K_5} U^{K_6} V^{K_7} \tag{7}$$

and since they must be dimensionless, the following must be true:

$$M^0L^0t^0 = (M^2L)^{K_1}\left(\frac{t}{M}\right)^{K_2}\left(\frac{M^3}{L}\right)^{K_3}(t^3)^{K_4}(L^2t)^{K_5}\left(\frac{L}{M^2t}\right)^{K_6}(ML^2t^2)^{K_7} \tag{8}$$

By combining units the following is true:

$$M^0L^0t^0 = M^{(2K_1-K_2+3K_3-2K_6+K_7)}L^{(K_1-K_3+2K_5+K_6+2K_7)}t^{(K_2+3K_4+K_5-K_6+2K_7)} \tag{9}$$

This results in

$$2K_1-K_2+3K_3-2K_6+K_7 = 0 \tag{10}$$

$$K_1-K_3+2K_5+K_6+2K_7 = 0 \tag{11}$$

$$K_2+3K_4+K_5-K_6+2K_7 = 0 \tag{12}$$

which are the homogeneous linear algebraic equations whose coefficients are the numbers in the rows of the dimensional matrix.

Now choose the repeating variables equal in number to the number of rows in the dimensional matrix. These variables are system parameters not including the dependent parameters. They should meet the following:

1. independent within themselves
2. easily controlled experimentally
3. contain all fundamental units at least once
4. not equal

Langhaar (Reference 10) chose K_5, K_6, and K_7. Solve the algebraic equations for the exponents of the chosen repeating variables -- K_5, K_6, and K_7. Starting with

$$
\begin{bmatrix}
2 & -1 & 3 & 0 & 0 & -2 & 1 \\
1 & 0 & -1 & 0 & 2 & 1 & 2 \\
0 & 1 & 0 & 3 & 1 & -1 & 2
\end{bmatrix}
\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \\ K_7 \end{Bmatrix} = 0 \tag{13}
$$

manipulating to

$$
\begin{bmatrix}
11 & -9 & 9 & -15 & 1 & 0 & 0 \\
-5 & 4 & -5 & 6 & 0 & 1 & 0 \\
-8 & 7 & -7 & 12 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \\ K_7 \end{Bmatrix} = 0 \tag{14}
$$

resulting in

$$K_5 = -11K_1 + 9K_2 - 9K_3 + 15K_4 \tag{15}$$

$$K_6 = 5K_1 - 4K_2 + 5K_3 - 6K_4 \tag{16}$$

$$K_7 = 8K_1 - 7K_2 + 7K_3 -12K_4 \tag{17}$$

Then assume values for K_1, K_2, K_3, and K_4 such that only one appears in each π term. This results in the Matrix of Solutions.

MATRIX OF SOLUTIONS

	P^{K_1}	Q^{K_2}	R^{K_3}	S^{K_4}	T^{K_5}	U^{K_6}	V^{K_7}
π_1	1	0	0	0	-11	5	8
π_2	0	1	0	0	9	-4	-7
π_3	0	0	1	0	-9	5	7
π_4	0	0	0	1	15	-6	-12

Write the π terms

$$\pi_1 = PT^{-11}U^5V^8 \tag{18}$$

$$\pi_2 = \frac{QT^9}{U^4V^7} \tag{19}$$

$$\pi_3 = \frac{RU^5V^7}{T^9} \tag{20}$$

$$\pi_4 = \frac{ST^{15}}{U^6V^{12}} \tag{21}$$

Therefore, if

$$\pi_{2_{model}} = \pi_{2_{prototype}} \tag{22}$$

$$\pi_{3_m} = \pi_{3_p} \tag{23}$$

$$\pi_{4_m} = \pi_{4_p} \tag{24}$$

then

$$\pi_{1_{model}} = \pi_{1_{prototype}} \tag{25}$$

This states that the prototype response P_p relates to the model response P_m as

$$P_p = \left(\frac{U_m{}^5 V_m{}^8 T_p{}^{11}}{U_p{}^5 V_p{}^8 T_m{}^{11}}\right) P_m \tag{26}$$

In the same manner, all other model-to-prototype relationships can be determined.

SECTION III

STANDARD SCALING DERIVATION

A. DISCUSSION

The vast majority of the work cited in the literature assumes that the same materials will be used in the model as in the prototype or that at least the material properties will be kept constant. Contingent on this assumption, a standard scaling table is often presented but never derived. A sound understanding of these standard scaling laws is required for any modelling study. It is therefore beneficial to derive and evaluate these laws and not take them at face value as done in other model studies. A common form of this scaling table is Table 1. It is essentially the same table presented in Nielson's (Reference 9) and Cheney's (Reference 12) work. It lists the relevant physical quantities from length to hydrodynamic time. All must be appropriately defined in a model study.

Material strength parameters are omitted because they must be constant between model and prototype for the table to be valid. Each of the scaling laws corresponding to its respective physical quantity is derived from an appropriate dimensionless product (π term). If all π terms were presented for each of these quantities, it would be apparent that material properties strongly influence the terms. Subsequently, researchers such as Sabnis (Reference 13), Morris (Reference 8), Nielson (Reference 9), and many others fix these properties and keep them constant in model and prototype. This causes

TABLE 1. STANDARD SCALING LAWS

QUANTITY	FULL SCALE (PROTOTYPE)	CENTRIFUGE MODEL AT n g's
LINEAR DIMENSION, (L)	1	1/n
GRAVITY (g)	1	n
AREA, (A)	1	$1/n^2$
VOLUME, (V)	1	$1/n^3$
DYNAMIC TIME, (t_d)	1	1/n
VELOCITY (DISTANCE/TIME), (v)	1	1
ACCELERATION (DISTANCE/TIME^2), (a)	1	n
DENSITY (MASS/VOLUME), (ρ)	1	1
UNIT WEIGHT (FORCE/UNIT VOLUME) (γ)	1	n
FORCE, (F)	1	$1/n^2$
STRESS (FORCE/AREA), (σ)	1	1
MASS, (M)	1	$1/n^3$
ENERGY, (E_n)	1	$1/n^3$
STRAIN (DISPLACEMENT/UNIT LENGTH), (ϵ)	1	1
HYDRODYNAMIC TIME, (t_h)	1	$1/n^2$
IMPULSE	1	$1/n^3$

the scale of length and scale of gravity to be the same. As Section III B indicates, this allows an accurate and simple formulation of scaling laws listed. Note that this derivation does not use π terms exclusively. Only the length scale and the gravity π term are needed. The remaining quantities fall out from common definition.

B. TABLE 1 DERIVATION

This section lists a step-by-step formulation of Table 1. Each scaling law derived is formulated based on previous laws or definitions. Note that the subscripts m and p correspond to model and prototype, respectively.

1. LENGTH (L)

By definition the scale factor, $S_L = \frac{L_p}{L_m}$

$$\therefore \quad L_m = \frac{L_p}{S_L} \qquad (27)$$

2. GRAVITY (g)

By using a π term which includes L and g, such as $\frac{L\,g\,\rho}{E}$ where E is modulus of elasticity, the following is true:

$$\frac{L_m g_m \rho_m}{E_m} = \frac{L_p g_p \rho_p}{E_p}$$

By assuming the same materials -- $E_m = E_p$ and $\rho_m = \rho_p$ the following is true:

$$g_m = \frac{L_p}{L_m} g_p \ , \quad \frac{L_p}{L_m} = S_L$$

$$\therefore \quad g_m = S_L g_p \qquad (28)$$

$$\therefore \quad S_L = n, \text{ where } n = \# \text{ of gravities.} \qquad (29)$$

3. AREA (A)

$$A_m = L_m^{\ 2} \ , \quad L_m = \frac{L_p}{n}$$

$$\therefore \quad A_m = \frac{L_p^{\ 2}}{n^2} = \frac{A_p}{n^2}$$

$$A_m = \frac{A_p}{n^2} \qquad (30)$$

4. VOLUME (V)

$$V_m = L_m^{\ 3} \ , \quad L_m = \frac{L_p}{n} \ , \ V_p = L_p^{\ 3}$$

$$\therefore \quad V_m = \frac{L_p^{\ 3}}{n^3} = \frac{V_p}{n^3}$$

$$V_m = \frac{V_p}{n^3} \qquad (31)$$

5. DYNAMIC TIME (t_d)

$$L = \tfrac{1}{2} g \ t_d^{\ 2} \quad \therefore \quad t_d = \frac{2L}{g}$$

$$L_m = \frac{L_p}{n} \ , \quad g_m = n g_p$$

$$\therefore \quad t_{d_m} = \frac{\frac{2L_p}{n}}{n g_p} = \frac{t_{d_p}}{n}$$

$$t_{d_m} = \frac{t_{d_p}}{n} \qquad (32)$$

6. VELOCITY (v)

$$v_m = L_m/t_m \ , \quad L_m = \frac{L_p}{n} \ , \quad t_m = \frac{t_p}{n}$$

$$\therefore \quad v_m = L_p/n/t_p/n = \frac{L_p}{t_p} = v_p$$

$$v_m = v_p \qquad (33)$$

7. ACCELERATION (a)

$$a_m = L_m/t_m^2 \ , \quad L_m = \frac{L_p}{n} \ , \quad t_m = \frac{t_p}{n}$$

$$\therefore \quad a_m = {}^{L_p/n}/{}_{t_p^2/n^2} = \frac{nL_p}{t_p^2} = na_p$$

$$a_m = na_p \qquad (34)$$

8. DENSITY (ρ) MASS/UNIT VOLUME

This is constant in all gravity environments.

$$\rho_m = \rho_p \qquad (35)$$

9. UNIT WEIGHT (γ) FORCE/UNIT VOLUME

$$\gamma_m = g_m \rho_m \ , \quad g_m = ng_p \ , \quad \rho_m = \rho$$

$$\therefore \quad \gamma_m = ng_p\rho_p = n\gamma_p$$

$$\gamma_m = n\gamma_p \qquad (36)$$

10. FORCE (F)

$$F_m = \gamma_m V_m \;, \quad \gamma_m = n\gamma_p \;, \quad V_m = \frac{V_p}{n^3}$$

$$\therefore \quad F_m = n\gamma_p V_p / n^3 = \frac{\gamma_p V_p}{n^2} = \frac{F_p}{n^2}$$

$$F_m = \frac{F_p}{n^2} \tag{37}$$

11. STRESS (σ)

$$\sigma_m = {}^{F_m}/_{A_m} , \quad F_m = {}^{F_p}/_{n^2} , \quad A_m = \frac{A_p}{n^2}$$

$$\therefore \quad \sigma_m = \frac{{}^{F_p}/_{n^2}}{{}^{A_p}/_{n^2}} = \frac{F_p}{A_p} = \sigma_p$$

$$\sigma_m = \sigma_p \tag{38}$$

12. MASS (M), with respect to dead load

$$M_m = {}^{F_m}/_{a_m} , \quad F_m = {}^{F_p}/_{n^2} , \quad a_m = na_p$$

$$\therefore \quad M_m \frac{{}^{F_p}/_{n^2}}{na_p} = \frac{F_p}{n^3 a_p} = \frac{M_p}{n^3}$$

$$M_m = \frac{M_p}{n^3} \tag{39}$$

13. ENERGY (E_n)

$$E_{n_m} = M_m c_m^{\,2} , \quad c_m = \frac{L_m}{t_m} , \quad M_m = \frac{M_p}{n^3}$$

$$L_m = \frac{L_p}{n} , \quad t_m = {}^{t_p}/_{n}$$

$$\therefore \quad E_{n_m} = \frac{M_p}{n^3} \; {}^{L_p/_n}/_{t_p/_n} = \frac{M_p L_p}{n^3 t_p} = \frac{E_{n_p}}{n^3}$$

$$E_{n_m} = \frac{E_{n_p}}{n^3} \qquad (40)$$

14. STRAIN (ε)

$$\varepsilon_m = \frac{\Delta L_m}{L_m} , \quad L_m = {}^{L_p}/_{n}$$

$$\therefore \quad \varepsilon_m = \frac{\Delta L_{p/n}}{L_{p/n}} = \frac{\Delta L_p}{L_p} = \varepsilon_p$$

$$\varepsilon_m = \varepsilon_p \qquad (41)$$

15. PERMEABILITY (k)

$$k_m = \frac{\upsilon_m}{i_m} , \quad i_m = \frac{\Delta h_m}{L_m} , \quad \upsilon_m = \upsilon_p$$

$$h_m = {}^{h_p}/_{n} , \quad L_m = L_{p/_n}$$

$$\therefore \quad k_m = \frac{\upsilon_p}{\Delta h_{p/_n}/L_{p/_n}} = \frac{\upsilon_p}{i_p} = k_p$$

$$k_m = k_p \qquad (42)$$

16. HYDRODYNAMIC TIME (t_h)

$$t_{h_m} = \frac{T_m (H_m^{\,2})}{C_{V_m}} \tag{43}$$

Assuming same material:

a) $T_m = T_p$ Time Factor

b) $C_{V_m} = C_{V_p}$ Coefficient of Consolidation

c) $H_m = H_{p/n}$ Drainage Path

$$\therefore\ t_{h_m} = \frac{T_p \dfrac{H_p^{\,2}}{n^2}}{C_{V_p}} = \frac{t_{h_p}}{n^2}$$

$$t_{H_m} = \frac{t_{H_p}}{n^2} \tag{44}$$

17. IMPULSE

$$I_m = F_m\, t_{d_m}\ ,\quad F_m = \frac{F_p}{n^2}\ ,\quad t_{d_m} = \frac{t_{d_p}}{n}\quad \therefore\ I_m = \frac{F_p}{n^2}\,\frac{t_{d_p}}{n} = \frac{I_p}{n^3}$$

$$I_m = \frac{I_p}{n^3} \tag{45}$$

SECTION IV

PISETS PROGRAM

To automate the π term generation procedure outlined in Section II, Theodore Self, a graduate student at the University of Florida, wrote a APL computer program called PISETS. This program was written to run on the Commodore Super PET microcomputer, using Waterloo Computing Systems micro APL version 1, and on the Northeast Regional Data Center system through the Virtual Machine/System Product Conversion Monitor System using VS APL. A computer user's manual and program listing are contained in Appendix.

The program requires the dimensional matrix and fundamental units a input. Using the procedure presented in Section II it generates the solution matrix for all or selected π sets. With system parameters numbering 10 or more the generation of the solution matrix takes a considerable effort. Noting that each set of independent repeating variables generates one π set and realizing that there are the number o system parameters factorial possible π sets, the use of PISETS is imperative.

Use of the program is limited only to the available computer internal working memory (RAM). Processing time is just a few seconds. Printed output is minimal unless a large dimensional matrix is used an all possible π sets are needed. Unfortuately, the standard SuperPET printer is not compatible with APL. The mainframe must be used to obtain a hard copy. Presently a 3 by 22 matrix has been run on the

above systems with no difficulty. Note that more than the standard three fundamental units can be used. If thermal or electrical units are needed in the study, PISETS will accommodate.

SECTION V

INITIAL PROBLEM SET-UP

A. IDENTIFICATION OF DEPENDENT AND SYSTEM PARAMETERS

The first step to the π term generation is identifying the dependent parameters (those to be evaluated in the study). The responses of interest are: stress (σ), displacements (d), and acceleration (a) due to an explosive loading. The structure is loaded with a point charge of energy (E_n) with a peak pressure P_o. The magnitude and characteristics of the dependent parameter on this energy input and an entire list of system parameters.

The list of system parameters should include all relevant material, geometric and energy variables that affect on the response parameters. Nielson (Reference 9) provided an initial list of system parameters from a similar study involving a concrete burster slab, Table 2. From the standpoint of the GLCM structure, additional parameters needed to be included. Neilson's effort did not include the material properties of steel reinforcement. Since the GLCM requires substantial steel reinforcing, steel strength, cross-sectional area and modulus were added to Nielson's list. Soil cohesion, angle of internal friction, preconsolidation pressure, and permeability are also added but are easily determined from Nielson's work by inspection. Additional parameters of strain and Poisson's ratio are added but only for clarification for they are π terms in themselves. Table 3 shows a complete list of the dependent and system parameters used in the initial effort.

TABLE 2. SYSTEM PARAMETERS FOR BLAST LOADING (Reference 9)

Parameter	Description	Dimension
Explosives	Energy, E	$ML^{2}T^{-2}$
	Pressure, P_0	$ML^{-1}T^{-2}$
	Depth of Burial, D	L
Burster-Slab	Thickness, H	L
	Mass Density, ρ_1	ML^{-3}
	Dilatational Wave Speed, C_1	LT^{-1}
	Poisson's Ratio, μ_1	--
	Strength, α_1	$ML^{-1}T^{-2}$
Soil (Sand)	Mass Density, ρ_1	ML^{-3}
	Dilatational Wave Speed, C_2	LT^{-1}
	Poisson's Ratio, μ_2	--
	Strength Parameter, α_2	$ML^{-1}T^{-2}$
Other Parameters	Gravity, g	LT^{-2}
	Bucket Dimensions, r, d	L
	Range	L

TABLE 3. LIST OF PROBLEM PARAMETERS

DEPENDENT PARAMETERS

1. σ - STRESS 2. d - DISPLACEMENT 3. a - ACCELERATION

SYSTEM PARAMETERS

1.	P_o	- CHARACTERISTIC PRESSURE	14.	E_s	- SOIL MODULUS
2.	I	- IMPULSE	15.	c	- SOIL COHESION
3.	E_n	- ENERGY	16.	P_c	- PRECONSOLIDATION PRESSURE
4.	R	- RADIUS	17.	k	- PERMEABILITY
5.	H	- THICKNESS	18.	g	- GRAVITY
6.	ρ_c	- CONCRETE MASS DENSITY	19.	T	- TIME
7.	C_c	- CONCRETE P-WAVE SPEED	20.	ϕ	- SOIL ANGLE OF FRICTION
8.	F_c'	- CONCRETE STRENGTH	21.	ε_{st}	- STEEL STRAIN
9.	E_c	- CONCRETE MODULUS	22.	ε_s	- SOIL STRAIN
10.	F_{st}	- STEEL STRENGTH	23.	ε_c	- CONCRETE STRAIN
11.	A_{st}	- AREA OF STEEL	24.	μ_{st}	- STEEL POISSON RATIO
12.	ρ_s	- SOIL MASS DENSITY	25.	μ_c	- CONCRETE POISSON RATIO
13.	C_s	- SOIL P-WAVE SPEED	26.	E_{st}	- STEEL MODULUS

B. SELECTION OF FUNDAMENTAL UNITS

The expression of the dependent and system parameters in the dimensional matrix must be in the form of fundamental units. These units are typically either mass, length, time (MLT) or force, length, time (FLT). The choice between each depends on familiarity of use and ease of π term operation. If the π terms generated using FLT are not easy to satisfy, changing to MLT may make the scaling laws easier to satisfy. Since previous work used MLT (Reference 9) and the fact that unit mass does not change in multiple-gravity environments, the MLT convention was chosen.

C. SELECTION OF REPEATING PARAMETERS

To solve the dimensional matrix, three repeating parameters must be chosen. These parameters must be selected from the list of system parameters. The dependent parameters cannot be used without sacrificing the solution. The repeating parameters must be independent within themselves to ensure independent solutions. They must contain all the fundamental units at least once, without being equal, in order to ensure dimensionless solutions. Most importantly, they must be easily controlled, experimentally. This allows simple parameter modification to meet scaling laws once derived. Based on these restrictions impulse (I), height (H), and concrete modulus (E_c) were chosen as the initial repeating parameters.

SECTION VI

PI SET DETERMINATION

A. DIMENSIONAL MATRIX

Based on the selection of dependent parameters, system parameters and fundamental units, a dimensional matrix was developed, Table 4. Note that parameters 23-26 of Table 3 are not in the dimensional matrix. These are either π terms in themselves or redundant due to inspection.

B. SOLUTION MATRIX AND π TERMS

With the dimensional matrix and repeating parameters (impulse, height, and modulus) identified in PISETS, the program generates the solution matrix, Table 5. This matrix reduces to 19 π terms as defined by Equation (5), Table 6. Note that most of the π terms are dependent on the material properties of the system materials. Contingent on the use of the same material in the model and prototype,the resulting scaling laws fall out quite easily.

C. EVALUATION OF π TERMS AND SCALING LAWS

The simplest way to organize an evaluation process is to group the generated π terms into functional groups relating their driving influence on the system. The π terms are thus separated into geometric, strength, explosive and impulse groups. In this way the impact of a violated π term on the problem can be readily identified. For instance, if a π term defining a strength relationship requires the modulus of

TABLE 4. TRIAL DIMENSIONAL MATRIX

FUNDAMENTAL UNITS	INDEPENDENT PARAMETERS						SYSTEM PARAMETERS															
	σ	d	a	P_o	I	E_n	R	H	ρ_c	C_c	F_c'	E_c	Γ_{ST}	A_{ST}	ρ_s	C_s	E_s	c	P_c	k	g	T
M	1	0	0	1	1	1	0	0	1	0	1	1	1	0	1	0	1	1	1	0	0	0
L	-1	1	1	1	1	2	1	1	-3	1	-1	-1	-1	2	-3	1	-1	-1	-1	1	1	0
T	-2	0	-2	-2	-1	-2	0	0	0	-1	-2	-2	-2	0	0	-1	-2	-2	-2	-1	-2	1

TABLE 5. TRIAL SOLUTION MATRIX

FUNDAMENTAL UNITS	INDEPENDENT PARAMETERS				SYSTEM PARAMETERS																	
	σ	d	a	P_o	I	E_n	R	H	ρ_c	C_c	F_c'	E_c	F_{ST}	A_{ST}	ρ_s	C_s	E_s	c	P_c	k	g	T
π_1	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
π_2	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
π_3	0	0	1	0	2	0	0	-5	0	0	0	-2	0	0	0	0	0	0	0	0	0	0
π π_4	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
T π_5	0	0	0	0	0	1	0	-3	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
E R π_6	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M π_7	0	0	0	0	-2	0	0	6	1	0	0	1	0	0	0	0	0	0	0	0	0	0
S π_8	0	0	0	0	1	0	0	-3	0	1	0	-1	0	0	0	0	0	0	0	0	0	0
π_9	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0
π_{10}	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0
π_{11}	0	0	0	0	0	0	0	-2	0	0	0	0	0	1	0	0	0	0	0	0	0	0
π_{12}	0	0	0	0	-2	0	0	6	0	0	0	1	0	0	1	0	0	0	0	0	0	0
π_{13}	0	0	0	0	1	0	0	-3	0	0	0	-1	0	0	0	1	0	0	0	0	0	0
π_{14}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0
π_{15}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0
π_{16}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0
π_{17}	0	0	0	0	1	0	0	-3	0	0	0	-1	0	0	0	0	0	0	0	1	0	0
π_{18}	0	0	0	0	2	0	0	-5	0	0	0	-2	0	0	0	0	0	0	0	0	1	0
π_{19}	0	0	0	0	-1	0	0	2	0	0	0	1	0	0	0	0	0	0	0	0	0	1
					*			*				*										

*Repeating Variables

TABLE 6. INITIAL GENERATED π TERMS

$$\pi_1 = \frac{\sigma}{E_c}$$

$$\pi_2 = \frac{d}{H}$$

$$\pi_3 = \frac{a\ I^2}{H^5\ E_c^{\ 2}}$$

$$\pi_4 = \frac{P_o}{E_c}$$

$$\pi_5 = \frac{\varepsilon_n}{H^3 E_c}$$

$$\pi_6 = \frac{R}{H}$$

$$\pi_7 = \frac{H^6\ \rho_c\ E_c}{I^2}$$

$$\pi_8 = \frac{I\ C_c}{H^3\ E_c}$$

$$\pi_9 = \frac{F'_c}{E_c}$$

$$\pi_{10} = \frac{F_{st}}{E_c}$$

$$\pi_{11} = \frac{A_{st}}{H^2}$$

$$\pi_{12} = \frac{H^6\ E_c\ \rho_s}{I^2}$$

$$\pi_{13} = \frac{I\ C_s}{H^3\ E_c}$$

$$\pi_{14} = \frac{E_s}{E_c}$$

$$\pi_{15} = \frac{c}{E_c}$$

$$\pi_{16} = \frac{P_c}{E_c}$$

$$\pi_{17} = \frac{Ik}{H^3\ E_c}$$

$$\pi_{18} = \frac{I^2 g}{H^5\ E_c^{\ 2}}$$

$$\pi_{19} = \frac{E_c\ T\ H^2}{I}$$

steel to be twice that of the prototype, its effect on other strength relationship is relatively identifiable. Table 7 indicates the appropriate grouping.

TABLE 7: π TERM GROUPS

GROUP	π TERMS
Geometric	π_2, π_6, π_{11}
Strength	π_9, π_{10}, π_{14}, π_{15}, π_{16}
Explosive	π_1, π_4, π_5
Impulse	π_3, π_7, π_8, π_{12} π_{13}, π_{17}, π_{18}, π_{19}

Once grouped, the actual scaling laws are formulated by equating model π term to prototype π term (Equation(4)), using the length scale factor n (Equations (27) and (28)),and cancelling out like material properties. The geometric group results in the following, where subscripts m and p are model and prototype, respectively:

$$\pi_2 = \frac{d}{H} \quad \therefore \quad \frac{d_p}{H_p} = \frac{d_m}{H_m}, \; d_m = \left(\frac{H_m}{H_p}\right) d_p$$

$$\therefore \quad d_m = \frac{d_p}{n} \qquad (46)$$

$$\pi_6 = \frac{R}{H} \quad \therefore \quad \frac{R_p}{H_p} = \frac{R_m}{H_m}, \; R_m = \left(\frac{H_m}{H_p}\right) R_p$$

$$\therefore \quad R_m = \frac{R_p}{n} \tag{47}$$

$$\pi_{11} = \frac{A_{ST}}{H^2} \quad \therefore \quad \frac{A_{ST_p}}{H_p^2} = \frac{A_{ST_m}}{H_m^2}, \; A_{ST_m} = \left(\frac{H_m^2}{H_p^2}\right) A_{ST_p}$$

$$\therefore \quad A_{ST_m} = \frac{A_{ST_p}}{n^2} \tag{48}$$

The strength groups results in the following:

$$\pi_9 = \frac{F_c'}{E_c} \quad \therefore \quad \frac{F_{c_p}'}{E_{c_p}} = \frac{F_{c_m}'}{E_{c_m}}, \; F_{c_m}' = \left(\frac{E_{c_m}}{E_{c_p}}\right) F_{c_p}'$$

$$\therefore \quad F_{c_m}' = F_{c_p}' \tag{49}$$

$$\pi_{10} = \frac{F_{ST}}{E_c} \quad \therefore \quad \frac{F_{ST_p}}{E_{c_p}} = \frac{F_{ST_m}}{E_{c_m}}, \; F_{ST_m} = \left(\frac{E_{c_m}}{E_{c_p}}\right) F_{ST_p}$$

$$\therefore \quad F_{ST_m} = F_{ST_p} \tag{50}$$

$$\pi_{14} = \frac{E_s}{E_c} \quad \therefore \quad \frac{E_{s_p}}{E_{c_p}} = \frac{E_{s_m}}{E_{c_m}}, \; E_{s_m} = \left(\frac{E_{c_m}}{E_{c_p}}\right) E_{s_p}$$

$$\therefore \quad E_{s_m} = E_{s_p} \qquad (51)$$

$$\pi_{15} = \frac{c}{E_c} \quad \therefore \quad \frac{c_p}{E_{c_p}} = \frac{c_m}{E_{c_m}}, \; c_m = \left(\frac{E_{c_m}}{E_{c_p}}\right) c_p$$

$$\therefore \quad c_m = c_p \qquad (52)$$

$$\pi_{16} = \frac{P_c}{E_c} \quad \therefore \quad \frac{P_{c_p}}{E_{c_p}} = \frac{P_{c_m}}{E_{c_m}}, \; P_{c_m} = \left(\frac{E_{c_m}}{E_{c_p}}\right) P_{c_p}$$

$$\therefore \quad P_{c_m} = P_{c_p} \qquad (53)$$

The explosive group results in the following:

$$\pi_1 = \frac{\sigma}{E_c} \quad \therefore \quad \frac{\sigma_p}{E_{c_p}} = \frac{\sigma_m}{E_{c_m}} \, , \; \sigma_m = \left(\frac{E_{c_m}}{E_{c_p}}\right) \sigma_p$$

$$\therefore \quad \sigma_m = \sigma_p \tag{54}$$

$$\pi_4 = \frac{P_o}{E_c} \quad \therefore \quad \frac{P_{o_p}}{E_{c_p}} = \frac{P_{o_m}}{E_{c_m}} \, , \; P_{o_m} = \left(\frac{E_{c_m}}{E_{c_p}}\right) P_{o_p}$$

$$\therefore \quad P_{o_m} = P_{o_p} \tag{55}$$

$$\pi_5 = \frac{E_n}{H^3 \, E_c} \quad \therefore \quad \frac{E_{n_p}}{H^3_p \, E_{c_p}} = \frac{E_{n_m}}{H^3_m \, E_{c_m}} \, , \; E_{n_m} = \left(\frac{H^3_m \, E_{c_m}}{H^3_p \, E_{c_p}}\right) E_{n_p}$$

$$\therefore \quad E_{n_m} = \frac{E_{n_p}}{n^3} \tag{56}$$

The impulse group results in the following:

$$\pi_3 = \frac{a\, I^2}{H^5\, E_c^{\,2}} \quad \therefore \quad \frac{a_p\, I_p^{\,2}}{H_p^5\, E_{c_p}^{\,2}} = \frac{a_m\, I_m^{\,2}}{H_m^5\, E_{c_m}^{2}}, \; a_m = \left(\frac{I_p^{\,2}}{I_m^2}\, \frac{H_m^5\, E_{c_m}^{\,2}}{H_p^5\, E_{c_p}^{2}}\right) a_p$$

$$\therefore \quad a_m = n\, a_p \qquad (57)$$

$$\pi_7 = \frac{H^6\, \rho_c\, E_c}{I^2} \quad \therefore \quad \frac{H_p^{\,6}\, \rho_{c_p}\, E_{c_p}}{I_p^{\,2}} = \frac{H_m^{\,6}\, \rho_{c_m}\, E_{c_m}}{I_m^{\,2}}, \; \rho_{c_m} = \left(\frac{H_p^{\,6}\, E_{c_p}\, I_m^{\,2}}{H_m^{\,6}\, E_{c_m}\, I_p^{\,2}}\right) \rho_{c_p}$$

$$\therefore \quad \rho_{c_m} = \rho_{c_p} \qquad (58)$$

$$\pi_8 = \frac{I\, C_c}{H^3\, E_c} \quad \therefore \quad \frac{I_p\, C_{c_p}}{H_p^3\, E_{c_p}} = \frac{I_m\, C_{c_m}}{H_m^3\, E_{c_m}}, \; C_{c_m} = \left(\frac{I_p\, H_m^3\, E_{c_m}}{I_m\, H_p^3\, E_{c_p}}\right) C_{c_p}$$

$$\therefore \quad C_{c_m} = C_{c_p} \qquad (59)$$

$$\pi_{12} = \frac{H^6 \; E_c \; \rho_s}{I^2} \quad \therefore \quad \frac{H_p^{\;6} \; E_{c_p} \; \rho_{s_p}}{I_p^{\;2}} = \frac{H_m^{\;6} \; E_{c_m} \; \rho_{s_m}}{I_m^{\;2}} \; , \; \rho_{s_m} = \left(\frac{H_p^{\;6} \; E_{c_p} \; I_m^{\;2}}{H_m^{\;6} \; E_{c_m} \; I_p^{\;2}}\right) \rho_{s_p}$$

$$\therefore \; \rho_{s_m} = \rho_{s_p} \qquad (60)$$

$$\pi_{13} = \frac{I \; C_s}{H^3 \; E_c} \quad \therefore \quad \frac{I_p \; C_{s_p}}{H_p^3 \; E_{c_p}} = \frac{I_m \; C_{s_m}}{H_m^3 \; E_{c_m}} \; , \; C_{s_m} = \left(\frac{I_p \; H_m^3 \; E_{c_m}}{I_m \; H_p^3 \; E_{c_p}}\right) C_{s_p}$$

$$\therefore \; C_{s_m} = C_{s_p} \qquad (61)$$

$$= \frac{I \; k}{H^3 \; E_c} \quad \therefore \quad \frac{I_p \; k_p}{H_p^3 \; E_{c_p}} = \frac{I_m \; k_m}{H_m^3 \; E_{c_m}} \; , \; k_m = \left(\frac{I_p \; H_m^3 \; E_{c_m}}{I_m \; H_p^3 \; E_{c_p}}\right) k_p$$

$$\therefore \; k_m = k_p \qquad (62)$$

$$\pi_{18} = \frac{I^2 g}{H^5 E_c^2} \quad \therefore \quad \frac{I_p^2 g_p}{H_p^5 E_{c_p}^2} = \frac{I_m^2 g_m}{H_m^5 E_{c_m}^2}, \; g_m = \left(\frac{I_p^2 H_m^5 E_{c_m}^2}{I_m^2 H_p^5 E_{c_p}^2}\right) g_p$$

$$\therefore \quad g_m = n \, g_p \qquad (63)$$

$$\pi_{19} = \frac{E_c T H^2}{I} \quad \therefore \quad \frac{E_{c_p} T_p H_p^2}{I_p} = \frac{E_{c_m} T_m H_m^2}{I_m}, \; T_m = \left(\frac{E_{c_p} T_m H_p^2}{E_{c_m} I_p H_m^2}\right) T_p$$

$$\therefore \quad T_m = \frac{T_p}{n} \qquad (64)$$

With the scaling laws established, the following scaling requirements must be determined:

1. Do the scaling laws agree with the standard scaling laws in Table 1?
2. Are they consistent with the assumption that the materials used in the prototype and model have the same properties?
3. Are the laws physically consistent and possible within themselves?

If all three of the conditions are reasonably satisfied, the π set can be used to govern the design and testing of a scale GLCM model.

This π set developed with impulse, height, and modulus of elasticity satisfies all three requirements. Equations (46), (47), (48), (53), (55), (56), (57), (63), and (64) are all consistent with Table 1. Equation (49), (50), (51), (52), (54), (58), (59), (60), (61), and (62) are consistent with the assumption that material properties between model and prototype are the same. It appears that all π terms are physically consistent as well; however, a re-evaluation of the system parameters indicates a more applicable π set may be available.

The present list of system parameters in the dimensional matrix consists of 19 quantities. Based on GLCM site definition and redundancy within the list, it is author's contention that only 17 are needed without losing clarity or applicability. The two parameters dropped are impulse and permeability. Impulse is a function of pressure, area, and time. Since all three are defined in the system parameter list, impulse is redundant. With regards to permeability, it is only relevant when water or some pore fluid is present. Since GLCM sites do not have swallow water tables, permeability is not a concern. Based on these considerations a new π set should be determined from the reduced system parameter list.

D. ALTERNATIVE π SET

By deleting impulse the previous choice of repeating parameters cannot be maintained. A new selection is required. This leads to a trial and error process, whereby the following methods can be used to find a reasonable π set.

1. Combine the existing π terms by multiplication or division.
2. Select new repeating variables

or

3. Change the system of fundamental unit (MLT or FLT)

With the existence of PISETS the simplest method is to select another set of repeating variables and simply re-evaluate the new π set. With each new selection, PISETS will check for independence and calculate a solution matrix. The operator must ensure that the other repeating variable requirements listed in Section II are met. Several new π sets were considered. The preferred π set is discussed in Section VII.

SECTION VII

FINAL π SET DETERMINATION

A. DIMENSIONAL MATRIX

The preferred π set for the GLCM shelter resulted from the selection of concrete modulus of elasticity (E_c), concrete mass density (ρ_c) and fundamental height (H) as the repeating variables. Table 8 indicates the dimensional matrix used in the final solution.

Note that impulse and permeability are no longer in the list of system parameters. Impulse was redundant, as discussed in Chapter VI, and was therefore deleted. Permeability was deleted, due to the unlikelihood of a high water table at potential GCLM shelter sites. This is often a favorable deletion because of the difficulty of realistically evaluating permeability in a centrifuge. The use of the same soil in model and prototype does not mean permeabilities will be the same. Although the pore passages will be the same, the driving forces are not; therefore neither is permeability. This will be further discussed later in this section.

B. SOLUTION MATRIX AND π TERMS

Inputting the dimensional matrix and identifying E_c, ρ_c, and H as the repeating variables in PISETS, the solution matrix is generated, Table 9.

TABLE 8. FINAL DIMENSIONAL MATRIX

FUNDAMENTAL UNITS	INDEPENDENT PARAMETERS						SYSTEM PARAMETERS													
	σ	d	a	P_o	E_n	R	H	ρ_c	C_c	F'_c	E_c	F_{ST}	A_{ST}	ρ_s	C_s	E_s	c	P_c	g	T
M	1	0	0	1	1	0	0	1	0	1	1	1	0	1	0	1	1	1	0	0
L	-1	1	1	-1	2	1	1	-3	1	-1	-1	-1	2	-3	1	-1	-1	-1	1	0
T	-2	0	-2	-2	-2	0	0	0	-1	-2	-2	-2	0	0	-1	-2	-2	-2	-2	1

TABLE 9. FINAL SOLUTION MATRIX

FUNDAMENTAL UNITS	INDEPENDENT PARAMETERS				SYSTEM PARAMETERS															
	σ	d	a	P_o	E_n	R	H	ρ_c	C_c	F_c'	E_c	F_{ST}	A_{ST}	ρ_s	C_s	E_s	c	P_c	g	T
π TERMS π_1	2	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0
π_2	0	2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
π_3	0	0	2	0	0	0	2	2	0	0	-2	0	0	0	0	0	0	0	0	0
π_4	0	0	0	2	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0
π_5	0	0	0	0	2	0	-6	0	0	0	-2	0	0	0	0	0	0	0	0	0
π_6	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
π_7	0	0	0	0	0	0	0	1	2	0	-1	0	0	0	0	0	0	0	0	0
π_8	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0
π_9	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0
π_{10}	0	0	0	0	0	0	-4	0	0	0	0	0	2	0	0	0	0	0	0	0
π_{11}	0	0	0	0	0	0	0	-2	0	0	0	0	0	2	0	0	0	0	0	0
π_{12}	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	2	0	0	0	0	0
π_{13}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	2	0	0	0	0
π_{14}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	2	0	0	0
π_{15}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	2	0	0
π_{16}	0	0	0	0	0	0	2	2	0	0	-2	0	0	0	0	0	0	0	2	0
π_{17}	0	0	0	0	0	0	-2*	-1*	0	0	1*	0	0	0	0	0	0	0	0	2

*Repeating Variables

Note that in comparison to Table 5, the diagonal number sequence across the matrix is 2 and not 1. This indicates that in generation of the homogeneous algebraic equation for the repeating variables similar to Equations (15), (16), and (17), fractions resulted in the exponents. PISETS does not operate with fractional exponents and therefore, multiplies the exponent appropriately in order to get an integer. In this case each term was squared. When the π terms are actually written, the square root can be taken to bring them back to the lowest set of integer exponents. For instance

$$\pi_1 = \frac{\sigma^2}{E_c^{\,2}} \qquad (65)$$

which reduces to

$$\pi_1 = \frac{\sigma}{E_c} \qquad (66)$$

while remaining dimensionless.

The matrix results in 17 π terms, two less than the initial trial matrix in Table 10. This reduction is consistent with Equation (5) and the reduction in system parameters.

C. EVALUATION OF π TERMS

As with trial π sets, this π set is grouped for evaluation. It is divided into geometric, strength, explosive and mass density groups, shown in Table 11.

TABLE 10. SOLUTION π TERMS

$$\pi_1 = \frac{\sigma}{E_c}$$

$$\pi_2 = \frac{d}{H}$$

$$\pi_3 = \frac{a\, H\, \rho_c}{E_c}$$

$$\pi_4 = \frac{P_o}{E_c}$$

$$\pi_5 = \frac{E_n}{H^3\, E_c}$$

$$\pi_6 = \frac{R}{H}$$

$$\pi_7 = \frac{\rho_c\, C_c^{\,2}}{E_c}$$

$$\pi_8 = \frac{F_c{}'}{E_c}$$

$$\pi_9 = \frac{F_{ST}}{E_c}$$

$$\pi_{10} = \frac{A_{ST}}{H^2}$$

$$\pi_{11} = \frac{\rho_s}{\rho_c}$$

$$\pi_{12} = \frac{\rho_c\, C_s^{\,2}}{E_c}$$

$$\pi_{13} = \frac{E_s}{E_c}$$

$$\pi_{14} = \frac{c}{E_c}$$

$$\pi_{15} = \frac{P_c}{E_c}$$

$$\pi_{16} = \frac{H\, \rho_c g}{E_c}$$

$$\pi_{17} = \frac{E_c\, T^2}{H^2\, \rho_c}$$

TABLE 11: SOLUTION π TERM GROUPS

GROUP	π TERMS
Geometric	π_2, π_6, π_{10}
Strength	π_8, π_9, π_{13}, π_{14}, π_{15}
Explosive	π_1, π_4, π_5
Mass Density	π_3, π_7, π_{11}, π_{12}, π_{16}, π_{17}

The groups are reduced to the scaling laws in the same way as the initial set (Section VI). The geometric group resulted in the following laws:

$$\pi_2 = \frac{d}{H} \qquad \therefore \frac{d_p}{H_p} = \frac{d_m}{H_m} \qquad \therefore d_m = \left(\frac{H_m}{H_p}\right) d_p$$

$$\therefore d_m = \frac{d_p}{n} \tag{67}$$

$$\pi_6 = \frac{R}{H} \qquad \therefore \frac{R_p}{H_p} = \frac{R_m}{H_m} \qquad \therefore R_m = \left(\frac{H_m}{H_p}\right) R_p$$

$$\therefore R_m = \frac{R_p}{n} \tag{68}$$

$$\pi_{10} = \frac{A_{ST}}{H^2} \quad \therefore \quad \frac{A_{ST_p}}{H^2_p} = \frac{A_{ST_m}}{H^2_m} \quad \therefore A_{ST_m} = \left(\frac{H^2_m}{H^2_p}\right) A_{ST_p}$$

$$\therefore \; A_{ST_m} = \frac{A_{ST_p}}{n^2} \qquad (69)$$

Since the strength group contains only material property ratios, using the same materials in model as prototype results in a one to one relationship. Therefore, the strength group dictates the following scaling laws:

$$F_c{'}_m = F_c{'}_p \qquad (70)$$

$$F_{ST_m} = F_{ST_p} \qquad (71)$$

$$E_{s_m} = E_{s_p} \qquad (72)$$

$$c_m = c_p \qquad (73)$$

$$P_{c_m} = P_{c_p} \qquad (74)$$

The explosive group reduces to three scaling laws.

$$\pi_1 = \frac{\sigma}{E_c} \quad \therefore \quad \frac{\sigma_p}{E_{c_p}} = \frac{\sigma_m}{E_{c_m}} \quad \therefore \; \sigma_m = \left(\frac{E_{c_m}}{E_{c_p}}\right) \sigma_p$$

$$\therefore \; \sigma_m = \sigma_p \qquad (75)$$

$$\pi_4 = \frac{P_o}{E_c} \quad \therefore \quad \frac{P_{o_p}}{E_{c_p}} = \frac{P_{o_m}}{E_{c_m}} \quad \therefore \quad P_{o_m} = \left(\frac{E_{c_m}}{E_{c_p}}\right) P_{o_p}$$

$$\therefore \quad P_{o_m} = P_{o_p} \qquad (76)$$

$$\pi_5 = \frac{E_n}{H^3 \; E_c} \quad \therefore \quad \frac{E_{n_p}}{H^3_p \; E_{c_p}} = \frac{E_{n_m}}{H^3_m \; E_{c_m}} \quad \therefore \quad E_{n_m} = \left(\frac{H^3_m E_{c_m}}{H^3_p \; E_{c_p}}\right) E_{n_p}$$

$$\therefore \; E_{n_m} = \frac{E_{n_p}}{n^3} \qquad (77)$$

The mass density group reduces as follows

$$\pi_7 = \frac{aH\rho_c}{E_c} \quad \therefore \quad \frac{a_p \; H_p \; \rho_{c_p}}{E_{c_p}} = \frac{a_m \; H_m \; \rho_{c_m}}{E_{c_m}} \quad \therefore \quad a_m = \left(\frac{H_p \; \rho_{c_p} \; E_{c_m}}{H_m \; \rho_{c_m} E_{c_p}}\right) a_p$$

$$\therefore \; a_m = n \; a_p \qquad (78)$$

$$\pi_7 = \frac{\rho_c \; C_c^{\,2}}{E_c} \quad \therefore \quad \frac{\rho_{c_p} \; C_{c_p}^{\,2}}{E_{c_p}} = \frac{\rho_{c_m} \; C_{c_m}^{\,2}}{E_{c_m}} \quad \therefore \quad C_{c_m} = \left(\frac{\rho_{c_p} \; E_{c_m}}{\rho_{c_m} \; E_{c_p}}\right)^{1/2} C_{c_p}$$

$$\therefore \quad C_{c_m} = C_{c_p} \qquad (79)$$

$$\pi_{11} = \frac{\rho_s}{\rho_c} \quad \therefore \quad \frac{\rho_{s_p}}{\rho_{c_p}} = \frac{\rho_{s_m}}{\rho_{c_m}} \quad \therefore \qquad \rho_{s_m} = \rho_{s_p} \qquad (80)$$

$$\pi_{12} = \frac{\rho_c \; C_s^{\,2}}{E_c} \quad \therefore \quad \frac{\rho_{c_p} \; C_{s_p}^{\,2}}{E_{c_p}} = \frac{\rho_{c_m} \; C_{s_m}^{\,2}}{E_{c_m}}$$

$$\therefore \quad C_{s_m} = \left(\frac{\rho_{c_p} \; E_{c_m}}{\rho_{c_m} \; E_{c_p}}\right)^{1/2} C_{s_p}$$

$$\therefore \quad C_{s_m} = C_{s_p} \qquad (81)$$

$$\pi_{16} = \frac{H \rho_c g}{E_c} \therefore \frac{H_p \rho_{c_p} g_p}{E_{c_p}} = \frac{H_m \rho_{c_m} g_m}{E_{c_m}}$$

$$\therefore g_m = \left(\frac{H_p \rho_{c_p} E_{c_m}}{H_m \rho_{c_m} E_{c_p}}\right) g_p$$

$$\therefore g_m = n g_p \tag{82}$$

$$\pi_{17} = \frac{E_c T^2}{H^2 \rho_c} \therefore \frac{E_{c_p} T^2_p}{H^2_p \rho_{c_p}} = \frac{E_{c_m} T^2_m}{H^2_m \rho_{c_m}}$$

$$\therefore T_m = \left(\frac{E_{c_p} H^2_m \rho_{c_m}}{E_{c_m} H^2_p \rho_{c_p}}\right)^{1/2} T_p$$

$$\therefore T_m = \frac{T_p}{n} \tag{83}$$

Once determined, the scaling laws are checked for consistency with the three scaling requirements listed in section 6.3

Equations (67), (68), (69), (75), (76), (77), (78), (79), (82) and (83) are all consistent with Table 1. They indicate that the standard scaling laws are not violated and, therefore, satisfy the first condition

for a workable π set. Equations (70), (71), (72), (73), (74), (79), and (81) are consistent with the use of the same material properties between model and prototype and satisfy the second condition for a viable π set. The last condition is more difficult to determine. Are the scaling laws physically possible to satisfy and maintain within themselves?

The answer to this question leads to several inconsistencies and conflicts between assumed properties and derived scaling laws. The following discussion does not invalidate this π set but qualifies its use. Note at the outset other researchers (References 8,14) have identified these problems and have either suggested solutions or dismissed the problems away.

The major problem stems from the internal conflict between satisfying the geometric scaling laws while maintaining the material properties. Just as the external length of the GLCM structure must be reduced by n, according to Equation 68, the internal dimensions of the concrete and soil must be as well. For soil, a reduced grain size drastically changes its material properties. For concrete, similar material properties pertubations occur (Reference 15). For concrete, an answer is provided by Sabnis and White (Reference 14), who show that gypsum mortar can be used instead of concrete on small-scale models. Their work indicates that prototype material properties are duplicated while satisfying geometric scale. Note that the gypsum particle may not exactly meet the scaled particle requirements but 10^{-7} meter size particles do not cause perturbations and therefore can be used. For soil, the solution is not as simple. Soil is a two-phase material consisting of solids and fluids. The various interactions of these phases make it impossible to change one property without affecting

others. If the particle size is reduced, all of the π terms relating soil modulus, mass density, and dilational wave speed change. It is appropriate to avoid these alterations by ignoring the geometric reduction for soil particles.

By using the same soil in model as in prototype consistent material properties can be assumed. Any question as to adverse effects on soil structure interaction due to this procedure was addressed by Morris (Reference 8). He indicates that if the model structure is sufficiently large with respect to soil particle size, the material properties can be maintained without significant effect on structure soil interaction. This sufficient size differential will change as the model changes. To determine the limiting differential, a modeling of models procedure should be conducted. This can be done by testing a generic model at different gravities and checking for inconsistencies. If this is done and the scale chosen accordingly, the same soil can be used without concern.

The secondary problem of the π set was alluded to in Section VII A. It is common in all centrifuge testing, using the same soil as prototype. It is often ignored as in the generation of this π set, but it needs to be clarified. The problem is permeability. On exposure to multiple-gravity forces, water will pass through soil pore passages faster than at one gravity. The hydrodynamic scaling law (Equation (44)) indicates this fact. When stresses build up in a structure/soil system in which water is present, the stresses in the structure will dissipate at n times faster than the prototype but the pore water pressure will dissipate at n^2 time faster. This negates similitude between model and prototype. It may be possible to introduce a more viscous fluid into

the problem to slow the dissipation, but this would require an evaluation of the corresponding changes in soil shear strength. It is sufficient to be aware of this problem and avoid it by keeping water tables out of the system.

Based on these assumptions and clarifications, the third requirement is satisfied. The scaling laws are physically consistent and possible within themselves. From the standpoint of internal consistency of the scaling laws and similitude, the stage is set for the model testing of the GLCM shelter.

SECTION VIII

PRACTICAL LIMITATIONS

A. LIMITATIONS

With the basic scaling laws established, the scale to which the GLCM model can actually be tested must be ascertained. The applicable scale to be used must be evaluated outside of the scaling laws and theory of multiple-gravity testing. The physical limitations of equipment, structure constructability, soil sample preparation, explosive, and instrumentation play a vital role in the determination of scale. Each of these considerations may or may not induce uncertainties into testing, depending on the scale and complexity of the problem. The GLCM shelter testing program is susceptible to limitations by all five considerations.

1. Equipment

The primary factor dictating the selected scale or size of the model is the availability of centrifuges built to test dynamic behavior. Presently, within the United States, eight existing centrifuges and two potential centrifuges are suitable for a GLCM testing program. Table 12 indicates their location and respective capabilities. The recommended model heights and lengths are based on Avgherinos' and Schofield's (Reference 16) work and Bassett's (Reference 17) work. Because of the radical nature of centrifugal accelerations applied to a model within a centrifuge, the gravitational forces on the model will vary across its

TABLE 12: AVAILABLE CENTRIFUGES

CENTRIFUGE	RADIUS (FT)	MAX. ACC. (g's)	MAX PAYLOAD (lbs)	MODEL * HEIGHT (in)	MODEL * LENGTH (in.)	REQUIRED ** SCALE
EXISTING						
New Mexico Engineering Research Institute	6.0	100	500	7.2	19	1/175***
University of California, Davis	3.3	175	57	4.0	11	1/303***
University of Maryland	4.42	200	100	5.3	14	1/238***
Sandia Labs	7	150	500	8.4	22	1/152
Princeton University	4.99	200	249	6.0	16	1/208
Boeing Aerospace	4.6	1200	122	5.5	15	1/222
University of Florida	3.3	100	50	4.0	11	1/303***
POTENTIAL						
University of Florida	6.6	160	185	8.0	21	1/159
Ames Lab	28	--	--	34	88	1/38

* These are theoretical values which do not consider actual physical limitations.
** This does not include limit on weight or spacing needed to reduce edge effects.
*** These machines are inadequate due to limit on maximum acceleration.

base and height, unlike that of the prototype. To limit this effect, Avgherinos and Schofield (Reference 16) suggest that the height of the model be limited to 1/10 of the radial arm of the centifuge. Bassett (Reference 17) suggests the length be no longer than the cord of a 15° section with a radius equal to the centrifuge's arm. The values provided are theoretical and do not reflect the actual physical requirements of the machine itself.

Aside from the acceleration variation, two additional equipment concerns affect the choice of scale. The scales suggested in Table 12 do not consider the weight limit associated with the maximum centrifugal accelerations. As the payload weight increases, the maximum available gravities decrease. If the eventual GLCM model weighs more than the specific weight limit, the corresponding decrease in acceleration must be calculated into the required scale. Secondly, there is no allowance for edge effects. As the explosive energy strikes the base or sides of the spinning platform, it reflects back into the model, disrupting the model response. Space should be allowed between the model and these surfaces to minimize these effects. Consequently, the actual scale may have to be smaller than indicated in Table 12. The actual space needed will depend on the energy absorbability of the spinning platform surfaces, the energy-transmission ability of the soil and the length of response-time histories required. A recommended spacing will have to be evaluated experimentally for each centrifuge used. An experiment using just a soil mass, loaded by a small explosive surface charge, should suffice in providing the information.

2. Constructability of Model Structures

To satisfy the scale needed to actually place a model in a centrifuge, the structure must be constructable. As previously discussed, gypsum mortar has been suggested by Sabnis and White (Reference 14) for the structural material. The extent to which it can be poured to a 1/100 and smaller scale is not known. For example, if the New Mexico Engineering Research Institute's (NMERI) centrifuge was used, the interior wall thickness of the GLCM shelter model would have to be at least 0.2 inches thick. When considering that steel reinforcement has to be placed within that wall, the difficulty of the problem becomes apparent. Although there is some research in scale model reinforcement (Reference 18) this effort will require an independent study. The walls are too small with respect to the amount of reinforcement required.

3. Model Soil Preparation

The prototype soil material is primarily granular. This significantly reduces the complexity of the model soil preparation. Size effects present no significant problem if Morris' (Reference 8) guidance (Section VII C) is adhered to. Consolidation effects, primarily associated with clay are essentially nonexistent. The previously discussed inconsistency with permeability is avoided, assuming water is excluded. Reconstruction of stability within the soil from chemical processes is of no concern since the prototype requires disturbed soil. Therefore, it is sufficient for testing to place the soil material at the required density and allow the appropriate stresses to develop during the actual testing of the model (Reference 19).

4. Explosives

By virtue of the scaling laws established, energy scales at $1/n^3$ that of the prototype energy. This requires very small energy sources for excitation of the model structure. For instance, the NMERI's centrifuge would need, at a minimum, a charge of .0007 pounds of TNT to match a 2,000-pound prototype threat. This is 1/100 of an ounce of TNT or equivalent explosive. For conventional threats of 500-pound to 2,000-pound bombs this could be very difficult. For nuclear threats the downsizing of the energy source would be less difficult due to several orders of magnitude increase in prototype source. Regardless, the equivalent explosive is quite small and presses the techrical limits of commercial explosives.

Nielson (Reference 9) has proposed that cyclothimethlenetrinitromine (RDX) be used for the above expressed purpose. Unfortunately, his efforts do not go down to the scales needed for the GLCM, Table 13. However, with a modification to a Reynold RP-83 detonator, he has recorded some reasonable data about the energy source itself. It may be possible to reduce this detonator even more for GLCM application.

It is generally appropriate to apply scaling laws to the explosive shape, as well to its energy output. Nielson suggests that the majority of the randomness in his data is attributable to explosive geometric variation and violation of the geometric scaling law of $1/n$. But since the GLCM's threat is projectile in nature and it is surrounded by a soil overburden, the geometric violations may not be a large consequence. If the peak pressure and duration of the explosive can be appropriately duplicated by the time it strikes the structure, the shape requirements may possibly be ignored. At present, it is unclear whether

this simplification or explosive can be used on a model GLCM shelter. If the proposed model testing is to be limited to the available centrifuges an explosive evaluation and development program will be required. Considering several potential threats to the GLCM shelter, this could be a costly development.

TABLE 13. THEORETICAL MODEL EXPLOSIVE SIMULATION WEIGHTS (Reference 9)

Threat Designation (lbs)	Centrifuge Environment (Gravities)				
	20 g	40 g	60 g	80 g	100 g
	Weight of RDX in grams				
250	12.35	1.54	0.46	0.19	0.10
500	25.57	3.20	0.95	0.40	0.20
1000	53.43	6.68	1.98	0.83	0.43
2000	106.95	13.37	3.98	1.67	0.86

5. Instrumentation

Like other limitations on physical equipment of a potential testing system, instrumentation is a consideration. With small-scaled structures, the placement of instrumentation cannot inhibit the structure's response. They must be small enough and be placed in such manner and number as to prevent perturbations in response. Fortunately the technology exists to meet the GLCM testing needs. Sabnis et al. (Reference 13) indicate that electrical resistance strain gages exist as small as 1/64 of an inch thick. Ryan's (Reference 20) work on piles used such gages with reasonable success.

For stresses, two possibilities exist, direct or indirect measurement. Nielson (Reference 9) suggests Dynasen carbon foil pressure sensors as a direct measure. Others suggest indirect measures using quartz transducers to measure velocity and then calculating stress by Equation (83).

$$\sigma = \rho c v \qquad (84)$$

where ρ = density, c = speed of light and v = measured velocity (Reference 21).

Other dynamic quantities such as acceleration and displacement can be measured using variations on the velocity quartz transducer. Complete specifications can be obtained through manufacturers listed in Table 14. Note: Although the gages exist for the quantities to be measured, caution should be used in the development of the instrumentation plan concerning space limitations for gage connections to external recording equipment.

TABLE 14. PARTIAL LIST OF INSTRUMENTATION MANUFACTURERS
(References 9, 13)

Manufacturer	Address	Type of Accessories
Acurex Corporation	485 Clyde Avenue Mountain View, CA 94042 Tel: (415) 964-3200	Data acquisition system for strains
BLH Electronics	42 Fourth Avenue Waltham, Mass. 02154 Tel: (617) 890-6700	All types of strain gages and related accessories for strain reading
Dynasen, Inc.	20 Dean Arnold Place Goleta, CA 93017	Pressure Sensors
Micromeasurements Div. of Vishay Inter-Technology, Inc.	P.O. Box 27777 Raleigh NC 27611 Tel: (919) 365-3800	All types of strain gages and related accessories for strain reading
PCB Peizotronics,	P.O. Box 33 Buffalo NY 14225 Tel: (716) 684-0001	Quartz Transducers for dynamic measurements
Strainsert Company	Union Hill Industrial Park West Conshocken, PA 19428	Flat load cells of various capacities

SECTION IX

RECOMMENDATIONS

A. SCALING LAWS

The technical and financial benefits of the centrifuge are set. The extent to which these benefits can be realized are not and are dependent on achieving similitude between the model and prototype. Using the scaling laws in Table 15, this dependency can be satisfied and a viable testing program can result. If adhered to, these scaling laws will creditably relate the GLCM model shelter response, point for point, to the prototype. These laws have been chosen to meet the standard scaling laws associated with common material properties, internal consistency and centrifuge capabilies; therefore, the extent to which the laws are satisfied will depend on the skill of researcher and the centrifuge chosen.

B. MATERIALS

Based on the available data and the considerable simplification of the modelling process, the materials used in model and prototype should have the same material properties.

Gypsum mortar is suggested to be used as a concrete substitute because it exhibits the appropriate material properties while avoiding aggregate scaling. The reinforcement should be of the same quality and strength as the prototype. Whereas soil quality and strength should be exactly the same as prototype to avoid material property

TABLE 15: RECOMMENDED SCALING LAWS

PARAMETER	SYMBOL	SCALING LAW	
1. Stress	σ	$\sigma_m = \sigma_p$	
2. Displacement	d		$d_m = \frac{d_p}{n}$
3. Acceleration	a	$a_m = n\ a_p$	
4. Velocity	v		$v_m = v_p$
5. Explosive Pressure	P_o	$P_{o_m} = P_{o_p}$	
6. Explosive Energy	E_n		$E_{n_m} = \frac{E_{n_p}}{n^3}$
7. Radius	R	$R_m = \frac{R_p}{n}$	
8. Thickness	H		$H_m = \frac{H_p}{n}$
9. Material Density	ρ	$\rho_m = \rho_p$	
10. Material Modulus	E		$E_m = E_p$
11. Material Strength	F	$F_m = F_p$	
12. Material Dilitional wave speed	C		$C_m = C_p$
13. Area	A	$A_m = \frac{A_p}{n^2}$	
14. Volume	V		$V_m = \frac{V_p}{n^3}$
15. Mass	M	$M_m = \frac{M_p}{n^3}$	
16. Material Strain	ε		$\varepsilon_m = \varepsilon_p$
17. Dynamic Time	t	$t_m = \frac{t_p}{n}$	
18. Material Poisson's Ratio	μ		$\mu_m = \mu_p$
19. Soil Cohesion	c	$c_m = c_p$	
20. Soil Preconsolidation Pressure	P_c		$P_{c_m} = P_{c_p}$
21. Force	F_F	$F_{F_m} = \frac{F_{F_p}}{n^2}$	
22. Soil Angle of Internal Friction	ϕ		$\phi_m = \phi_p$

inconsistencies. Any deviation from this recommendation will cause numerous inconsistencies in the scaling laws and several physical impossibilities in material behavior and experimental capabilities. The sources of uncertainty due to equipment and model construction are numerous enough without introducing material property variations. To avoid the inconsistency of the rate of structure and pore pressure stress dissipation due to changes in permeability, water should be left out of the problem. Assuming a low water table at potential GLCM sites, this restriction is consistent with the prototype.

C. CENTRIFUGE

The centrifuge chosen for the GLCM shelter model testing controls the scale to be chosen for testing. It is beneficial to the success of the program to choose the largest centrifuge available. As the scale gets larger the complexity and likelihood of experimental error reduces due to material and equipment limitations. The author believes that the complexity of the GLCM requires the use of the national centrifuge research facility at Ames Laboratory. This could possibly limit the scale reduction to 1/38. The use of any smaller facility will require extensive simplication of the model design due to model construction problems.

D. EXPLOSIVES

The availability of controllable explosives of the sizes needed for this model study is limited at best. Nielson's (Reference 9) work using RDA (cyclothimethlenetrinitromine) is encouraging but not conclusive. If large centrifuges similar to Ames facility are not available, a research program to develop an energy source small enough to be consistent with GLCM threats should be conducted.

E. INSTRUMENTATION

Instrumentation to measure stress, strain, and dynamic quantities is available commercially. The exact gages will have to be determined once the centrifuge and scale are determined. The number of gages per model will depend on the recording capability of the facility and wiring capabilities of the testing technician. The selection of gages should come from manufacturers listed in Table 14.

F. ASPECTS TO BE EVALUATED

Before actually testing the potential GLCM shelter model, several aspects of the testing setup should be evaluated. To adequately establish the testing program, a series of initial tests must be done to answer the following questions:

1. To what extent must measures be taken to limit edge effects?
2. To what scale must we limit the testing to avoid excessive perturbations on the test data from inappropriate soil structure interaction caused by unscaled soil grain size?
3. What is the optimum explosive charge shape to use?
4. To what extent do strain rate effects disturb this particular structure?
5. To what detail can we build the GLCM shelter model, based on physical construction limitations?

G. MODELLING OF MODELS

After the questions have been answered and scale set and testing procedure have been determined, the setup should be exposed to a modelling of models. This will not require a model for each gravity environment selected. Generic setups can be used to check consistency

of developed procedures. A series of environments such as 20, 40, 60, 80, and 100 gravities should indicate any inconsistency of the testing procedure due to the multiple-gravity loadings.

H. CONSTITUTIVE MODELLING

Because of scaling limitations (GLCM shelter size and capabilities of existing U.S. centrifuges, and uncertainties concerning edge effects, strain rate, instrumentation, and explosive shape and composition) multiple-gravity loadings of GLCM shelter components are recommended. These test results should be analyzed,using available numeric computer codes to ascertain compatibility between computer constitutive modelling and centrifugal modelling. If acceptable computer predictions of observed centrifugal model events can be made, then centrifugal modelling of the entire GLCM shelter may not be required.

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APPENDIX A

PISETS

An Interactive APL Workspace
For Use with Dimensional Analysis

User's Manual

TED SELF, 8/27/83

CONTENTS

PISETS

An Interactive APL Workspace
For Use With Dimensional Analysis

User's Manual

A. INTRODUCTION

The PISETS workspace contains functions which are used to generate all possible sets of nondimensional parameters for a given problem in dimensional analysis. If the generation of all sets is not desirable, a single set or group of sets may be specified.

The user must supply as input the total number of basic dimensions used and the total number of dimensioned parameters used. The user must also develop and input the dimensional matrix that specifies which basic dimensions appear in each dimensioned parameter.

Output will be in the form of a matrix of exponents which will indicate to what power each dimensioned parameter must be raised, prior to multiplication, to form each nondimensional parameter. An exponent matrix will be generated for each set of nondimensional parameters.

PISETS was implemented on the Commodore SuperPET microcomputer using Waterloo Computing System microAPL version 1.0.

PISETS was implemented on the Northeast Regional Data Center (NERDC) system through the Virtual Machine/System Product Conversational Monitor System (VM/SP CMS) using VS APL (APL*PLUS/I140).

Commands to be entered by the user will be indicated in **boldface** throughout this manual.

B. THE DIMENSIONAL MATRIX

A dimensional matrix for the problem should be developed prior to the use of PISETS.

Each column of the dimensional matrix corresponds to one of the dimensioned parameters; each row corresponds to one of the basic dimensions. The values in the matrix represent the power to which each basic dimension appears in each dimensioned parameter.

As an example, consider a problem for which three basic dimensions are chosen; Mass (M), Length (L) and Time (T). Assume that the problem (a spring-mass-dashpot system) is fully described by the seven dimensioned parameters, with their component basic dimensions, shown below:

y,	displacement,	L
y_0,	initial displacement,	L
m,	mass,	M
c,	damping coefficient,	MT^{-1}
k,	spring constant,	MT^{-2}
v_0,	initial velocity,	LT^{-1}
t,	time,	T

The dimensioned parameters are formed from powers of the basic dimensions. These exponents are entered into the dimensional matrix, as shown below:

	1 y	2 m	3 c	4 k	5 v_0	6 y_0	7 t
L	1	0	0	0	1	1	0
T	0	0	-1	-2	-1	0	1
M	0	1	1	1	0	0	0

It is important to record the column number that corresponds to each dimensioned parameter. When the exponent matrix is output each column of that matrix will correspond to the same dimensioned parameter that the same column of the dimensional matrix corresponds to.

C. HOW TO START AND STOP

1. The Commodore SuperPET Microcomputer

Turn on both the SuperPET and the disk drives and set the SuperPET's PROG:6502:6809 switch to 6809 and PROG:R/W:READ switch to R/W. The screen will then display the Waterloo microSystem menu.

Put the Waterloo System diskette in disk drive 1 and the PISETS diskette in disk drive 0.

From the Waterloo microSystem menu, select APL by entering the command:

apl

After a minute or so, the screen will display a message similar to the following:

WATERLOO MICRO APL VERSION 1.0 81/09/01

COPYRIGHT 1981 BY WATERLOO COMPUTING SYSTEMS LIMITED

CLEAR WS

Load the PISET workspace by entering the command:

> LOAD PISETS

The screen will respond with the date and time that the workspace was last saved. The PISET workspace is now ready for use.

To stop, enter the command:

> OFF

and remove the disks from the disk drives.

2) VS APL under NERDC

An example of the start of a VS APL terminal session is shown on page 82.

A terminal with the APL character set is required when using VS APL. If a deckwriter is used you will have hardcopy of your output. To get the terminal's attention you must push the BREAK key and then carriage return. Often, this must be done repeatedly.

The terminal will eventually respond with the prompt:

enter class

Respond by entering:

nerdc

If NERDC is available the terminal will print a system menu (do not select 1 for APL, it is a different system) from which vm/cms may be selected by entering:

v

The terminal will print:

vm/370 online

Push carriage return and enter the logon command as follows:

logon xxxxxxxx,yy

where xxxxxxxx represents your access number and yy is your sequence number. The terminal will respond with:

ENTER PASSWORD:

At this point you must type in your password.

Next a block of times and messages will be printed ending with:

NERDC VM/SP3 CMS

indicating that you are now in the CMS system but not yet in VS APL.

To access APL enter the command:

apl

A block of statistics will be printed followed by:

APL*PLUS/1140

If a CONTINUE workspace was saved previously, it will now be loaded by the system and the date and time that this workspace was saved will be printed. The CONTINUE workspace is a PISETS workspace that has been saved with the > CONTINUE command. This workspace is now ready for use.

If there is no CONTINUE workspace in the workspace library the terminal will print:

CLEAR WS

This will require that the command:

> LOAD PISETS

be entered, following which the terminal will print the date and time that the PISETS workspace was saved and PISETS will be ready for use.

To quit either > OFF or > CONTINUE may be used.

If the command:

> OFF

is entered then the present workspace contents will not be saved.

If the command:

> CONTINUE

is entered then the present workspace contents will be saved in the CONTINUE workspace and the previous contents of CONTINUE will be lost.

D. USING THE WORKSPACE

The APL functions INPUT, SET and CALC have been defined in the PISETS workspace and are used to input the dimensional matrix, choose

which sets of non-dimensional parameters will be output and to output the matrix of exponents for each of these sets.

The prompt > : indicates that numeric input is required from the terminal and the prompt GO: indicates that the function has been completed.

The function INPUT is used to set up the problem. An example using the spring-mass-dashpot system developed earlier is shown on page 83.

Entering the command:

INPUT

activates the function. In the example INPUT first requests the number of basic dimensions (three) and then the number of dimensioned parameters (seven).

The function then computes and prints the maximum number of sets of nondimensional parameters which may exist. This is an upper limit and not all combinations will produce a set of parameters. In the spring-mass-dashpot example there are 35 possible sets but only 24 sets actually exist.

The function now asks that the dimensional matrix be input. The matrix is input row by row, one value at a time. Care must be taken when entering negative numbers. The negative sign is not the same symbol as the subtraction symbol. The APL negative sign symbol is a shifted (uppercase) 2.

Entering the command:

CALC

activates the APL function CALC which will calculate and output an exponent matrix for each set of nondimensional parameters. Unless the function, SET, is used immediately prior to the function CALC all of the exponents matrices that exist will be output at once.

The complete output for all sets of nondimensional parameters in the spring-mass-dashpot example is listed on pages 86 to 89.

Entering command:

SET

activates the APL function SET which allows a choice of which set or sets of nondimensional parameters are output by function CALC.

The example on page 85 shows how SET is used to choose one particular set of nondimensional parameters. The example on page 86 shows how set is used to generate several "adjacent" sets of non-dimensional parameters.

The words "PI TERM SETS" refer to the Buckingham Pi-Theorem and are synonymous with "nondimensional parameter sets".

The user is given the choice of entering ALL, 1 or Range.

If the word ALL or the letter A is entered then the next use of CALC will cause all the sets of nondimensional parameters to be output. This is identical to the results of using CALC immediately after using either INPUT or CALC.

If the number 1 is entered, as shown in on page 84, then the next use of CALC will result in the output of only the chosen set of non-dimensional parameters. The user chooses the nondimensional parameter set by choosing which of the dimensioned parameters will be considered independent. These are the parameters which will be fixed (eliminated) when computing the values in the exponent matrix. The user enters the column numbers from the dimensional matrix, in ascending order, these correspond to the chosen dimensioned parameters.

If the word RANGE or the letter R is entered, as shown on page 85, then the user must choose a second set of dimensioned parameters to be

fixed. The next use of CALC will result in the output of a range of nondimensional parameter sets beginning with the first set chosen and ending with the second set chosen and including all sets "in between." The numeral formed by the column numbers of the second set must be "lower" than the numeral formed by the column numbers of the first set, for example 3 4 6 = 345 > 257 = 2 5 7.

E. SAVING A WORKSPACE

For large problems it may not be desirable to print out all possible nondimensional parameter sets in one sitting. It is also not desirable to have to input the dimensional matrix every time PISETS is used.

When an APL workspace is saved with the > SAVE command all of the variables which have been defined in that workspace are saved along with the functions. When this workspace is later loaded with the > LOAD command the values of these variables will be retained and need not be reinput.

After INPUT has been used to set up a problem, the workspace may be saved with the command:

> SAVE PISETS

and when PISETS is next loaded, work on that problem can continue without reuse of the function INPUT.

If it is desirable to save the input from more than one problem then the workspace for each problem may be saved under a different name. The command:

> WSID NEWNAME

will change the present workspace name to NEWNAME. The workspace may then be saved with the command:

> SAVE NEWNAME

When using VS APL under NERDC the > SAVE command will only work if a NEWNAME workspace already exists in the workspace library. If NEWNAME is not in the library then the > PSAVE command must be used in place of the > SAVE command.

The NEWNAME workspace may then be substituted for the PISETS workspace when desired.

When using NERDC, tne sign-off command > CONTINUE causes the present workspace to be renamed CONTINUE and saved under that name.

The command:

> LIB

will list the names of all the workspaces which have been saved in the workspace library.

F. THE EXPONENT MATRIX

The exponent matrix, which is obtained for each set of nondimensional parameters, determines how the dimensioned parameters are combined to form the nondimensional parameters.

Each row of the matrix corresponds to a nondimensional parameter and each column corresponds to a dimensioned parameter. Each nondimensional parameter is formed by raising each dimensioned parameter to the pcwer shown in its column and then multiplying all of the dimensioned parameters together.

Considering the spring-mass-dashpot example and the nondimensional parameter set that is generated on page 84. The resulting exponent matrix is interpreted as shown below:

		y	m	c	k	v_0	y_0	t		
PI1	=	1	0	0	0	0	-1	0	$= y^1 * m^0 * c^0 * k^0 * v_0^0 * y_0^{-1} * t^0$	$= y/y_0$
PI2	=	0	-1	1	0	0	0	1	$= y^0 * m^{-1} * c^1 I k^0 * v_0^0 * y_0^0 * t^1$	$= ct/m$
PI3	=	0	-1	0	1	0	0	2	$= y^0 * m^{-1} * c^1 * k^0 * v_0^0 * y_0^0 * t^2$	$= kt^2/m$
PI4	=	0	0	0	0	1	-1	1	$= y^0 * m^0 c^0 * k^0 * v_0^1 * y_0^{-1} * t^1$	$= v_0 t/y_0$

The four terms PI1, PI2, PI3, and PI4 are the nondimensional terms (pi terms) in this set. The problem may therefore be described by an equation of the form PI1 = f(PI2, PI3, PI4) or $y/y_0 = f(ct/m, kt^2/m, v_0 t/y_0)$.

Typical NERDC Sign-on Procedure

```
enter class nerdc
class nerdc start

Enter t for TCP, c for CICS, v for vm/cms, m for MUSIC, 1 for APL o for T

v
vm/370 online
!

logon 100600843,1
ENTER PASSWORD:
XXXXXXX
LOGMSG - 07:10:39 EDT WEDNESDAY 08/24/83
* A new release of VM/SP is now in production.  Please type
* "NEW" for information on changes in the system.  JHB
LOGAN AT 16:27:18 EDT WEDNESDAY 08/24/83
NERDC VM/SP3 CMS
apl
Z (1A0) R/O
X (1A1) R/O
W (1A2) R/O
U (1A3) R/O
V (1A4) R/O
R; T-0.10/0.21 16:27:26

    APL* PLUS/1140

SAVED 16:20:55 08/24/83
```

Function INPUT
Input for Spring-Mass-Dashpot Example

```
     INPUT

ENTER THE NUMBER OF BASIC DIMENSIONS USED
>:
      3
ENTER THE NUMBER OF DIMENSIONED PARAMETERS USED
>:
      7
THERE ARE AT MOST 35 SETS OF PI TERMS
ENTER 21 VALUES FOR THE DIMENSIONAL MATRIX BY ROWS
FOR ROW 1 ENTER COLUMN 1
>:
      1
FOR ROW 1 ENTER COLUMN 2
>:
      0
FOR ROW 1 ENTER COLUMN 3
>:
      0
FOR ROW 1 ENTER COLUMN 4
>:
      0
FOR ROW 1 ENTER COLUMN 5
>:
      1
FOR ROW 1 ENTER COLUMN 6
>:
      1
FOR ROW 1 ENTER COLUMN 7
>:
      0
FOR ROW 2 ENTER COLUMN 1
>:
      0
FOR ROW 2 ENTER COLUMN 2
>:
```

ETC. FOR EACH VALUE

```
FOR ROW 3 ENTER COLUMN 6
>:
      0
FOR ROW 3 ENTER COLUMN 7
>:
      0
GO:
```

Function SET - Function CALC
Single Set of Nondimensional Terms

```
    SET

HOW MANY PI TERM SETS DO YOU WANT TO SEE?
ENTER 'ALL', '1', OR 'RANGE'
1
ENTER THE COLUMN NUMBERS OF THE PARAMETERS TO BE FIXED (ELIMINATED)
ENTER 3 VALUES SEPARATED BY SPACES
>:
      2 6 7
GO:
      CALC

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 6 7
THE MATRIX OF EXPONENTS IS

      1   0   0   0   0  -1   0
      0  -1   1   0   0   0   1
      0  -1   0   1   0   0   2
      0   0   0   0   1  -1   1

GO:
```

Function SET - Function CALC
Multiple Sets of Nondimensional Terms

```
    SET

HOW MANY PI TERM SETS DO YOU WANT TO SEE?
ENTER 'ALL', '1', OR 'RANGE'
RANGE
ENTER THE COLUMN NUMBERS OF THE PARAMETERS TO BE FIXED (ELIMINATED)
ENTER 3 VALUES SEPARATED BY SPACES
>:
      3 4 6
ENTER THE COLUMN NUMBERS OF LAST SET OF PARAMETERS
ENTER 3 VALUES SEPARATED BY SPACES
>:
      2 5 7
GO:
      CALC

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 4 6
THE MATRIX OF EXPONENTS IS

      1   0   0   0   0  -1   0
      0   1  -2   1   0   0   0
      0   0   1  -1   1  -1   0
      0   0  -1   1   0   0   1

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 4 5
THE MATRIX OF EXPONENTS IS

      1   0  -1   1  -1   0   0
      0   1  -2   1   0   0   0
      0   0  -1   1  -1   1   0
      0   0  -1   1   0   0   1

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 6 7
THE MATRIX OF EXPONENTS IS

      1   0   0   0   0  -1   0
      0  -1   1   0   0   0   1
      0  -1   0   1   0   0   2
      0   0   0   0   1  -1   1

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 5 7
THE MATRIX OF EXPONENTS IS

      1   0   0   0  -1   0  -1
      0  -1   1   0   0   0   1
      0  -1   0   1   0   0   2
      0   0   0   0  -1   1  -1

GO:
```

```
CALC

PARAMETERS 5 6 7 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 4 6 7
THE MATRIX OF EXPONENTS IS
     1   0   0   0   0  -1   0
     0   1   0  -1   0   0  -2
     0   0   1  -1   0   0  -1
     0   0   0   0   1  -1   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 4 5 7
THE MATRIX OF EXPONENTS IS
     1   0   0   0  -1   0  -1
     0   1   0  -1   0   0  -2
     0   0   1  -1   0   0  -1
     0   0   0   0  -1   1  -1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 4 5 6
THE MATRIX OF EXPONENTS IS
     1   0   0   0   0  -1   0
     0   1   0  -1   2  -2   0
     0   0   1  -1   1  -1   0
     0   0   0   0   1  -1   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 6 7
THE MATRIX OF EXPONENTS IS
     1   0   0   0   0  -1   0
     0   1  -1   0   0   0  -1
     0   0  -1   1   0   0   1
     0   0   0   0   1  -1   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 5 7
THE MATRIX OF EXPONENTS IS
     1   0   0   0  -1   0  -1
     0   1  -1   0   0   0  -1
     0   0  -1   1   0   0   1
     0   0   0   0  -1   1  -1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 5 6
THE MATRIX OF EXPONENTS IS
     1   0   0   0   0  -1   0
     0   1  -1   0   1  -1   0
     0   0  -1   1  -1   1   0
     0   0   0   0   1  -1   1

PARAMETERS 3 4 7 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 4 6
THE MATRIX OF EXPONENTS IS
     1   0   0   0   0  -1   0
     0   1  -2   1   0   0   0
     0   0   1  -1   1  -1   0
     0   0  -1   1   0   0   1
```

```
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 3 4 5
THE MATRIX OF EXPONENTS IS
      1   0  -1   1  -1   0   0
      0   1  -2   1   0   0   0
      0   0  -1   1  -1   1   0
      0   0  -1   1   0   0   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 6 7
THE MATRIX OF EXPONENTS IS
      1   0   0   0   0  -1   0
      0  -1   1   0   0   0   1
      0  -1   0   1   0   0   2
      0   0   0   0   1  -1   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 5 7
THE MATRIX OF EXPONENTS IS
      1   0   0   0  -1   0  -1
      0  -1   1   0   0   0   1
      0  -1   0   1   0   0   2
      0   0   0   0  -1   1  -1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 5 6
THE MATRIX OF EXPONENTS IS
      1   0   0   0   0  -1   0
      0  -1   1   0  -1   1   0
      0  -1   0   1  -2   2   0
      0   0   0   0   1  -1   1

PARAMETERS 2 4 7 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 4 6
THE MATRIX OF EXPONENTS IS
      2   0   0   0   0  -2   0
      0  -1   2  -1   0   0   0
      0   1   0  -1   2  -2   0
      0  -1   0   1   0   0   2
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 4 5
THE MATRIX OF EXPONENTS IS
      2  -1   0   1  -2   0   0
      0  -1   2  -1   0   0   0
      0  -1   0   1  -2   2   0
      0  -1   0   1   0   0   2

PARAMETERS 2 3 7 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 3 6
THE MATRIX OF EXPONENTS IS
      1   0   0   0   0  -1   0
      0   1  -2   1   0   0   0
      0   1  -1   0   1  -1   0
      0  -1   1   0   0   0   1
```

```
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 2 3 5
THE MATRIX OF EXPONENTS IS
       1  -1   1   0  -1   0   0
       0   1  -2   1   0   0   0
       0  -1   1   0  -1   1   0
       0  -1   1   0   0   0   1

PARAMETERS 2 3 4 FORM MATRIX WITH ZERO DETERMINANT

PARAMETERS 1 6 7 FORM MATRIX WITH ZERO DETERMINANT

PARAMETERS 1 5 7 FORM MATRIX WITH ZERO DETERMINANT

PARAMETERS 1 5 6 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 4 7
THE MATRIX OF EXPONENTS IS
       0   1   0  -1   0   0  -2
       0   0   1  -1   0   0  -1
      -1   0   0   0   1   0   1
      -1   0   0   0   0   1   0

PARAMETERS 1 4 6 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 4 5
THE MATRIX OF EXPONENTS IS
      -2   1   0  -1   2   0   0
      -1   0   1  -1   1   0   0
      -1   0   0   0   0   1   0
      -1   0   0   0   1   0   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 3 7
THE MATRIX OF EXPONENTS IS
       0   1  -1   0   0   0  -1
       0   0  -1   1   0   0   1
      -1   0   0   0   1   0   1
      -1   0   0   0   0   1   0

PARAMETERS 1 3 6 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 3 5
THE MATRIX OF EXPONENTS IS
      -1   1  -1   0   1   0   0
       1   0  -1   1  -1   0   0
      -1   0   0   0   0   1   0
      -1   0   0   0   1   0   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 3 4
THE MATRIX OF EXPONENTS IS
       0   1  -2   1   0   0   0
      -1   0   1  -1   1   0   0
      -1   0   0   0   0   1   0
       0   0  -1   1   0   0   1
```

```
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 2 7
THE MATRIX OF EXPONENTS IS
       0  -1   1   0   0   0   1
       0  -1   0   1   0   0   2
      -1   0   0   0   1   0   1
      -1   0   0   0   0   1   0

PARAMETERS 1 2 6 FORM MATRIX WITH ZERO DETERMINANT

UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 2 5
THE MATRIX OF EXPONENTS IS
       1  -1   1   0  -1   0   0
       2  -1   0   1  -2   0   0
      -1   0   0   0   0   1   0
      -1   0   0   0   1   0   1
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 2 4
THE MATRIX OF EXPONENTS IS
       0  -1   2  -1   0   0   0
      -2   1   0  -1   2   0   0
      -2   0   0   0   0   2   0
       0  -1   0   1   0   0   2
UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS 1 2 3
THE MATRIX OF EXPONENTS IS
       0   1  -2   1   0   0   0
      -1   1  -1   0   1   0   0
      -1   0   0   0   0   1   0
       0  -1   1   0   0   0   1

GO:
```

G. PISETS PROGRAM LISTING

```
      ∇INPUT [⎕]∇
    ∇ INPUT
[1]   'ENTER THE NUMBER OF BASIC DIMENSIONS USED'
[2]   D←⎕
[3]   'ENTER THE NUMBER OF DIMENSIONED PARAMETERS USED'
[4]   Q←⎕
[5]   A←(D,Q)⍴0
[6]   IQ←⍳Q
[7]   PI←Q-D
[8]   'THERE ARE AT MOST ',(⍕(!Q)÷(!D)×(!PI)),' SETS OF PI TERMS'
[9]   'ENTER ',(⍕D×Q),' VALUES FOR THE DIMENSIONAL MATRIX BY ROWS'
[10]  ENTD←0
[11] INCROW:
[12]  ENTD←ENTD+1
[13]  →(ENTD>D)/GO
[14]  ENTQ←0
[15] INCCOL:
[16]  ENTQ←ENTQ+1
[17]  'FOR ROW ',(⍕ENTD),' ENTER COLUMN ',(⍕ENTQ)
[18]  A[ENTD;ENTQ]←⎕
[19]  →(ENTQ<Q)/INCCOL
[20]  →INCROW
[21] GO:'GO:'
[22]  STOP←(2*Q)-1
[23]  BASE←POWERS←0,⌽2*(⍳D)-1
    ∇
```

```
      ∇SET [⎕]∇
    ∇ SET
[1]   'HOW MANY PI TERM SETS DO YOU WANT TO SEE?'
[2]  ENTER:
[3]   'ENTER ''ALL'', ''1'', OR ''RANGE'''
[4]   CHOICE←1⍴⍞
[5]   →(CHOICE='A','1','R')/ALL,FIRST,FIRST
[6]   →ENTER
[7]  FIRST:
[8]   'ENTER THE COLUMN NUMBERS OF THE PARAMETERS TO BE FIXED
[9]   'ENTER ',(⍕D),' VALUES SEPERATED BY SPACES'
[10]  POWERS←⎕
[11]  STOP←+/POWERS←0,2*Q-POWERS
[12]  →(CHOICE='1')/END
[13]  'ENTER THE COLUMN NUMBERS OF THE LAST SET OF PARAMETERS'
[14]  'ENTER ',(⍕D),' VALUES SEPERATED BY SPACES'
[15]  STOP←⎕
[16]  STOP←+/2*Q-STOP
[17]  →END
[18] ALL:
[19]  POWERS←0,⌽2*(⍳D)-1
[20]  STOP←(2*Q)-1
[21] END:'GO:'
    ∇
```

LISTING CONTINUED

```
      ∇DET [⎕]∇
    ∇ Z←DET F;V;N;J
[1]   N←(⍴F)[Z←1]
[2]   V←0,(N−1)⍴1
[3]   J←(|F[;1])⍳⌈/|F[;1]
[4]   →(1=J)/7
[5]   F[1,J;]←F[J,1;]
[6]   Z←−Z
[7]   →(1E¯10<|F[1;1])/10
[8]   Z←0
[9]   →RETURN
[10]  Z←Z×F[1;1]
[11]  F[1;]←F[1;]÷F[1;1]
[12]  F←F−F[;1]∘.×F[1;]
[13]  F←V/[1] V/F
[14]  →(0<⍴V←¯1↓V)/3
[15] RETURN:
    ∇
```

```
      ∇CALC [⎕]∇
    ∇ CALC
[1]  ENCODE:' '
[2]   →((+/POWERS)>STOP)/END
[3]   ID←((Q⍴2)⊤+/POWERS)/IQ
[4]   →((|Z←DET A[;ID])<0.0001)/ZERO
[5]   'UNKNOWNS ELIMINATED CORRESPOND TO DIMENSIONED PARAMETERS ',(⍕ID
[6]   NID←(~IQ∊ID)/IQ
[7]   E←(Q,PI)⍴0
[8]   E[ID;]←(⌹A[;ID])+.×(−A[;NID])
[9]   E[NID;]←(PI,PI)⍴1,PI⍴0
[10]  'THE MATRIX OF EXPONENTS IS '
[11]  ⍉((|E)>0.01)×E×|Z
[12]  →NEXT
[13] ZERO:
[14]  'PARAMETERS ',(⍕ID),' FORM MATRIX WITH ZERO DETERMINANT'
[15] NEXT:
[16]  N←D+1
[17] LOOP:
[18]  POWERS[N]←2×POWERS[N]
[19]  →(POWERS[N]≥POWERS[N−1])/ENCODE
[20]  POWERS[N]←BASE[N]
[21]  N←N−1
[22]  →LOOP
[23] END:'GO:'
[24]  POWERS←BASE
[25]  STOP←(2*Q)−1
    ∇
```